

# Skewness and Kurtosis, Real Options, and Investment Under Uncertainty

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## Abstract

This study examines how the probability of exercising a real option is influenced by the skewness and kurtosis, in addition to volatility, of output price changes in the presence of sunk costs and irreversible investments. The underlying motivation is a modified Real Option Valuation model that accounts for skewness and kurtosis when the underlying price change distribution is not Normal. Oil price changes exhibit both volatility, non-Normal skewness and kurtosis. I therefore study several real choices made by oil well operators. Using a large sample of oil wells (53 million well-month observations) from five oil producing U.S. states, I examine the drilling, production and shut down decisions associated with each well during the period 2010-2019. Results based upon the estimation of panel choice models and Cox Proportional Hazard models, show that the decisions made are influenced by both skewness, kurtosis, volatility and expected prices as value maximizing theory in the presence of price uncertainty and sunk costs predict. The results are not consistent with a prediction that the personal preferences of the decision makers (aside from value maximization) influence the choices studied. The results are robust to a number of alternative tests and specifications. A one standard deviation increase in skewness is associated with a 13.10% increase in the probability to close a well; a similar increase in kurtosis leads to an 11.41% increase in the likelihood of drilling a new well, compared with volatility's impacts of -44.56% and -33.59%, respectively.

JEL Classification: G11 G13

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## 1. Introduction

Output price change uncertainty and sunk costs play a central role in modern theories of investment choice and the valuation and exercise of real options (Dixit (1989, 1991)) and Dixit and Pindyck (1994)). These features can, in theory, produce incentives to delay profitable investments and the abandonment of unprofitable investments. A typical assumption is that output price changes have a Normal distribution, but this is not necessarily always true.

A case in point is the price of oil. In recent years, we have witnessed wide swings in oil prices. Oil price changes exhibit high volatility (Ferderer (1996)) but also non-zero skewness and excess kurtosis, which are inconsistent with price changes described by a Gaussian/Normal distribution. However, the Dixit (Dixit (1989, 1991)) and Dixit and Pindyck (1994)) derivations of the relationship between price uncertainty and investment trigger prices relies on the assumption that price uncertainty can be described entirely by volatility. With many existing studies (Dai et al. (2021)) demonstrating that there exists non-zero skewness in oil prices, which I confirm along with the presence of excess kurtosis, this calls for a revisiting the empirical relations between investment and production decisions and the moments of the price change distribution, accounting for the higher order moments. That is, does the valuation and optimal exercise of real options depend on skewness and kurtosis in addition to volatility. Instead, an analysis based upon an extended option pricing model is called for.

In theory, option values can be impacted by the skewness and kurtosis of the price change distribution (Jarrow and Rudd (1982), Corrado and Su (1996a, 1996b), and Brown and Robinson (2002)). Oil exploration and production is one sector where there can be substantial uncertainty surrounding future oil prices and significant sunk costs of investment and that price changes exhibit non-Normal behavior. Thus the investment and production behavior of firms operating in this sector provides an ideal setting for studying the impact of expected skewness and kurtosis on those decisions.

The primary question I ask and answer is: Are investment and production choices influenced by the expected skewness and kurtosis of the price change distribution as well as volatility? I accomplish this task through the specification and estimation of discrete choice models utilizing a comprehensive sample of over 53 million well-month observations of the micro-level oil well drilling and operation decisions of operators in five major oil producing states. Specifically, I test whether the oil well production decisions are affected by oil price distribution moments - skewness and kurtosis, in this study. This study develops and empirically estimates, discrete choice models of the decision to drill and complete an oil well, to produce oil, to temporarily cease production (mothball) and to abandon (shut-in) that address the specific questions of how and to what extent skewness and kurtosis of the oil price change distribution, estimated

with the model-free option-implied risk-neutral central moments using the methods in Bakshi, Madan, and Kapadia (2003), affect the choices we observe in addition to volatility and sunk costs.

The set of decisions I examine in this study are tailor made for addressing the question of whether skewness and kurtosis of the price change distribution influence real investment and operating decisions. The detailed monthly investment and production records that form the basis for the study, the fact that the product produced is identifiable along with the expected output price and price change distribution moments, and the availability of detailed information about the characteristics of the investments, provide the ability to conduct a micro-level investigation of the choices to invest and produce generally not possible. The utility of such data has not gone unnoticed, having formed the basis of the investigations by Kellogg (2014), Gilje (2019), Anderson et al. (2018), and Décaire et al. (2020).

Oil and gas producers tend to expand drilling and completion activity only when prices are very high, and at the same time do not rapidly close or plug wells when prices are below the marginal cost of production. This research is the first study, to the author's knowledge, to investigate the implications of non-normality of output prices on the actual investment and production decisions of firms.

In addition to a number of controls, I specifically examine whether the main results are impacted by 1) the presence of sunk costs, 2) the influence of the productivity of nearby wells, 3) the regulatory environment of the state in which a well is located, and 4) whether the increased intensity of focus on Environmental, Social and Governance issues has impacted the choices observed and the relations between those choices and skewness and kurtosis. The mood created by the 'Environmental, Social, and Governance' (ESG) movement has had what many believe to be a profound impact on business decisions (see for instance the website maintained by the World Bank, <https://datatopics.worldbank.org/esg/>). No industry has been more impacted than the energy sector, and within that sector oil and natural gas producers. In this study, I also examine how the increased attention to ESG marked with the Paris Climate Agreement impacted the decisions studied.

The data set I examine allows me to investigate the generality of my findings through a study of oil producers' investment and production decisions in multiple states of the United States and geographically different production areas (basins). I use oil well drilling, shutdown, and production records from California, Pennsylvania, Texas, North Dakota, and Oklahoma to study the effects of price uncertainty on investment and production activity. While a company's quarterly or yearly capital expenditures provide a glimpse of capital spending decisions, a deeper understanding requires disaggregated micro-level data observed at a higher frequency. There are multiple benefits to using these data to explore the impact of uncertainty on investment choices. First, the data are measured monthly and provide information on new investment (drilling), disinvestment (abandonment) choices, and shut-in (producing or mothballing) choices. Second, a primary uncertainty these companies face is price

uncertainty reflected in the distributional characteristics of oil prices. Third, the data set examined allows a comparison of choices across different geographical locations and states. Fourth, the data allow an investigation of whether public companies behave differently than private companies. Fifth, the data allow me to examine the impact of an important shift brought on by the increased focus on 'Environmental, Social and Governance' (ESG) issues triggered by the adoption of the Paris Climate Accord in 2015. The data set examined includes upwards of 53 million observations that span ten years and multiple geographic areas and states of the United States.

My analysis involves the estimation of multiple panel choice models. The statistical approaches involve the estimation of Cox proportional hazard models structured to investigate the choice to drill and shut-down (abandon) and dynamic panel probit models to investigate the choice to continue producing or temporarily stop production. Implied moments of the oil price change distribution are central to the investigation and are model-free estimates computed from options on oil futures prices following the methods developed by Bakshi et al. (2003). My analysis reveals that higher-order moments (expected skewness and kurtosis of the price change distribution) are important determinants of oil producers' shut-in, investment, and disinvestment decisions for oil wells in California, North Dakota, Oklahoma, Pennsylvania, and Texas from 2010 to 2019. The results suggest that the option-implied higher-order moments proxy for jump risks or crash risks in output prices (oil futures prices) and leptokurtic risk. These characteristics have a significant impact on the propensity to choose to switch between "producing" or "mothballing". These characteristics also are significant in explaining the choice to shut down a well permanently. The expected price and future price change volatility are significant determinants of each of the decisions evaluated and their impact on the probability of choice is in the direction predicted by models of value maximizing choice. My results show that when option-implied oil price skewness increases by one standard deviation, the propensity to exercise a real option for irreversible investment decisions (reopen a mothballed oil well) increases by 1.75%. When implied kurtosis in oil prices increases by one standard deviation, investing likelihood increases by 5.38%. The propensity to immediately close an oil well increase by 13.10% when skewness increases by one standard deviation and the propensity to drill a new well increases by 11.41% when kurtosis increases by one standard deviation. Finally, the results I present are more consistent with decision makers following value maximizing decision rules than imposing their own person investor preferences for skewness and kurtosis.

In Section 2, I review the literature on real option valuation (ROV) and ROV's applications in the case of irreversible investment decisions. An economic model of investment choice is presented in Section 3 and develops hypotheses on the relationship between real options exercise and implied skewness and kurtosis. In Section 4 an econometric framework is developed for testing the hypotheses

and the data are discussed. Section 5 presents empirical results and discussions. Section 6 presents a summary and conclusions.

## **2. Related Literature**

### **2.1 Investment Under Uncertainty**

There are three important characteristics of many real investment decisions. First, investment is wholly or partially irreversible. That is investment involves sunk costs. Second, investment involves risky payoff streams. Third, new information arrives as time passes which can influence expectations about future risky payoffs. And fourth, most investment opportunities do not necessarily vanish if not taken up immediately. That is the investment decision involves not only whether to invest but also when.

The study of real investment activity and the theory of optimal investment choice by firms has a long and rich history (see Caballero (1999); Girardi (2021) and the references therein). The foundations of the optimal decision rule for investment choices by firms stem significantly from the Fisher Separation Theorem (Fisher, 1930) and Hirshleifer (1958, 1970), while the study of such choices under uncertainty was significantly impacted by the work of Tobin and Brainard (1976) and Hayashi (1982). The implications of sunk costs and future uncertainty play an important role in what many would call the modern theory of investment under uncertainty which brings to the fore the value of real options embedded in investment and production opportunities.

### **2.2 Irreversible Investment: Sunk Costs**

Sunk costs play an important role in the modern theory of real investment, especially in the presence of future uncertainty regarding investment payoffs due to future output price change uncertainty. Sunk costs of investment have been explored by Bernanke (1983), Pindyck (1990), Dixit and Pindyck (1994), Carruth et al. (2000), Holland et al. (2000), Kogan (2001), Cooper (2006), and Davis and Cairns (2017). Such costs are not reversible or recoverable. This characteristic is crucial when irreversible investments incur high sunk costs—for instance, mining and capital investments. For example, Hausman (1999) discusses the impact of sunk costs in telecommunication services investments. He finds that ignoring sunk costs and irreversibility leads to underinvestment in the telecommunication market. That regulation on price (the TSLRIC Standard) does not consider the critical fact that telecommunication services investments are irreversible and delay innovation in the industry. The two crucial questions regarding irreversible investment decisions are: first, whether to invest or abandon the option; second, when is the optimal time to invest, i.e., the optimal investment timing. Baldwin (1982) and Pindyck (1991) investigate the former problem and show investment rules for irreversible projects. As for the second question on irreversible investment, McDonald and Siegel (1986), Majd and Pindyck (1987), and Dixit

(1991) explores the problem of “when to invest” when projects are irreversible. Due to its nature of irreversibility, choosing whether to invest and the optimal time to invest is a crucial issue.

ROV is a valuable tool in solving these questions. ROV (Myers (1977)) solves the optimal timing problem of irreversible investment by taking the investment option as analogous to a financial option: the choice can be exercised at the expense of the investment cost (option premium) to obtain the cash flows from the investment project (option payoffs). Whether exercising the option depends on real options value and project value that jointly determine optimal exercise times. If investing is not feasible to the end of the investment horizon (option maturity), ROV suggests not investing. ROV explains the irreversible investment decisions by showing that it is optimal to exercise the “investment option” when the option value to wait does not exceed the value of immediate exercise rather than when project NPV exceeds zero (Dixit and Pindyck (1994)). I will discuss the real options (hereafter *RO*) exercises in more detail in the following section.

### **2.3 Price Change Volatility and Investment**

The negative relationship between price change volatility and the exercise of a real option to either invest or divest lies at the heart of the developments in the theory of real options (Dixit and Pindyck (1994)). The choice problem is intimately related to the presence of sunk costs of investment. It is the essential prediction of ROV compared with NPV when investment is not reversible. While NPV does not implicitly indicate the relationship between uncertainty and investment but the price risk and investment (Samis et al. 2005), ROV suggests that uncertainty leads to postponement of investment (Bloom et al. (2007)). The underlying well-known principle is that an option’s value increases as expected volatility increases, *ceteris paribus*. ROV predicts that more significant price uncertainty leads to an increase in real options value and postponement of options exercise (Bernanke (1983) and Abel, et al. (1996)). As the value of the option to wait increases with greater forward asset value volatility if the value of the option exceeds the value gained from immediate exercise the exercise will be postponed. Dixit and Pindyck (1994) conclude that more significant uncertainty increases the “waiting option” value and the propensity to postpone investment. In addition, they show that the threshold at which it becomes optimal to invest deviates from the Marshallian solution and increases with price uncertainty.

The relationship between price uncertainty and investment choices has been the theme of numerous studies focusing on other (than oil) industries and more general industry sectors. For instance, Drakos and Konstantinou (2013) present the connection of price uncertainty to investment decisions in the manufacturing industry. Doshi, Kumar, and Yerramilli (2018) show that the tendency to follow the real options exercising rules depends on the size of the firms through their examination of capital investment decisions. They find that larger firms are less likely to follow these rules as they mainly hedge

output price risk and can avoid the detrimental price movements – a price uncertainty. Other studies in real options include Pindyck (1993) on the effects of cost and technology uncertainty on real options investment, Aizenman and Marion (1993a, 1993b) on the impacts of policy and macroeconomic uncertainty on investments, and Lensink and Morrissey (2000) who examines the investment-uncertainty relationship in the aggregate level; the studies examine in firm-level include Leahy and Whited (1995) as well as Kang et al. (2014) and Gulen and Ion (2016) both examine the influences of policy uncertainty on investment; Jens (2017) who discuss political uncertainty effects.<sup>1</sup>

The studies on the relationship between price uncertainty and oil project investments reveal that higher uncertainty is associated with the postponement of oil drilling decisions. They show that oil price uncertainty is a critical determinant of whether to invest as well as investment timing for oil projects. Using oil well production data from Oklahoma and Texas, Molls (2001) shows that uncertainty is significantly related to the likelihood of oil wells “producing” status. Kellogg (2014) finds a negative association between uncertainty and the propensity to exercise real options in oil drilling. Décaire et al. (2020) reveal the Peer Exercise Behavior – the propensity for oil producers to decide whether to drill a new oil well is influenced by drilling decisions made in neighboring areas, suggesting that information spillover is important and that this information helps reduce information asymmetry when facing investment projects with sunk costs and price uncertainty.

Dixit and Pindyck (1994) also study the relationship between price uncertainty and the propensity to divest and find that in theory greater output price uncertainty can lead to the postponement of investment but also divestment. Moel and Tufano (2002) find that gold mining’s entry and exit options exercising behaviors are negatively related to uncertainty. Moel and Tufano (2002) extend the real options literature to the application in exits. That is, price uncertainty does not only postpone investment plans but also delays the exercise of divestment. Investors are reluctant to close an operating project when facing greater uncertainty in product output prices.

The “Bad News Principle” proposed by Bernanke (1983) suggests an asymmetry in the association between uncertainty and the probability of exercising a real option. Dixit and Pindyck (1994) show that more significant price uncertainty increases the optimal trigger price at which a real option will be exercised. The “Bad News Principle” suggests that only the downside risk potential of price matters. A mean-preserving increase in price uncertainty leads to higher expected “bad news” and postponement of a real option’s exercise while increasing the value of the option to wait.

## **2.4 Skewness, Kurtosis and Investment**

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<sup>1</sup> Gulen and Ion (2016) find evidence of a negative relationship between firm capital investment and policy uncertainty captured by a news-based index.

While the study of skewness and kurtosis in financial security returns has a long history (Conrad et al. (2013)), the literature by and large has ignored the implications of skewness and kurtosis of an asset's price change distribution on real investment activity, choosing to focus exclusively on either the expected price and or expected volatility. This is despite the fact as described earlier that higher order moments can influence the value of an option and consequently when it is optimal to exercise that option. One exception is the study by Schneider and Spalt (2016), which shows the relationship between project return skewness and investment decisions but not in the frame of ROV. Boyarchenko and Levendorskiĭ (2002) and Boyarchenko (2004) derive the optimal exercising rules of RO with jump-diffusion price process and suggest that their model potentially indicates the skewness-kurtosis and investment relationship.<sup>2</sup> Bernanke's theoretical development of his "Bad News Principal" (Bernanke (1983)) indicates that only adverse price shocks matter in the optimal timing of real investment. Thus, changes in skewness can be an indicator of the probability of "bad news" and result in a lower propensity to exercise a real investment option. Another critical higher central moment is kurtosis which I show in Section 4 can be a determinant of an option's value when the price change distribution is not a Normal distribution. I return to a discussion of option pricing adjusting for skewness and kurtosis in Section 3.

To my best knowledge, there is the first research exploring the relationship between irreversible investments real options exercise and higher-order central moments of price probability distributions. This study extends the study on the relationship between uncertainty and investment to higher-order moments of price.

### 3. Skewness, Kurtosis, and Real Options Investment

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<sup>2</sup> A separate literature has focused on the implications of skewness and kurtosis of financial security returns. One strand of the literature reveals that the relationship between equity investment and probability distribution skewness and kurtosis depends on forms of investor utilities (Brunnermeier and Parker (2004)). For example, Kraus and Litzenberger (1976) show investors' preference for positive skewness in the equity market, suggesting a non-quadratic form of investor utility. Another strand of studies explains the influences of skewness and kurtosis on investor preferences. For instance, Harvey and Siddique (2000) find that investors require a positive premium for systematic skewness, and Boyer et al. (2010) find a negative return premium for idiosyncratic skewness. While the previously mentioned studies use central return moments, others focus on the risk-neutral moments of returns. For instance, Conrad et al. (2013) reveal that more positive risk-neutral skewness is strongly related to lower subsequent returns. Bali and Murray (2013) show that risk-neutral skewness is negatively related to the equity portfolio returns. Both studies suggest investors' tendency to hold assets with positive skewness. Furthermore, Bali and Murray (2013) show that, although both sides of skewness influence asset returns, left-hand side skewness is remarkably priced into skewness assets by investors. Bali and Murray (2013) point out that exposure to skewness can be asymmetric by decomposing the left- and right-hand sides of skewness risk. Studies on implied higher moments and security price include Datta et al. (2017). Kurtosis is strongly related to investors' preferences and returns premiums. Related studies include Dittmar (2002) focus on the investor's preference on kurtosis, and other studies like Arouri and Nguyen (2010), Diavatopoulos et al. (2012), and Bachmann and Bayer (2014) explore the relationship of investment and both skewness and kurtosis.



Dixit and Pindyck (1994) explains the basic frames for solving the optimal price trigger problem under the Real Options Valuation method, in which investment projects with large sunk costs are considered to be embedded with the “delay” option to postpone investment decisions to next decision period. This “delay” option enables the postponement of investment decisions that allows better information on price distribution to be collected in the next decision period, possibly achieving greater project outcomes. However, an important underlying assumption in that analysis is that relative price changes are only a function of an expected price change and a normally distributed error, specifically Geometric Brownian Motion:

$$dP/P = \alpha dt + \sigma dz$$

Where  $\alpha$  is the drift parameter and  $\sigma$  is the variance parameter, which fully describe the distribution of the price change process. A key result of the analysis the identification of a ‘price’ trigger for investment or divestment that is a function of volatility. The Brownian Motion assumption does not deal with the case in which price changes exhibit skewness and kurtosis. Although studies including Boyarchenko and Levendorskiĭ (2002) discussed the solution for trigger investment prices and optimal exercising times with Levy process output prices and suggested the effects of skewness and kurtosis on the solution outcomes, no studies so far, to my best knowledge, considers the explicit relationship between skewness, kurtosis, and real options solutions. That is, the relationship between the higher-order moments of a non-normally distributed output prices and the optimal investment triggering prices has not been revealed. This study aims at solving this problem and filling this research gap.

An unique but rich dataset becomes adopted by more recent studies in the Real Options Valuation. The oil well drilling and closing activities and crude oil prices. The company’s capital investment data has a relatively lower observation frequency – most capital investments are reported on a quarterly basis and some are even reported on an annual basis. This low frequency of reporting limits the usage of company’s capital investments data in achieving the goal of examining the effects of prices of output products on investment choices. The company-level data however is unable to obtain a direct observation of its output prices. The output product’s prices, as well as its risk-neutral central moments, cannot be observed directly in financial markets. Instead, I use a relatively new but useful dataset in the commodity markets – the oil well drilling, production, and closing records and the WTI (West Texas Intermediate) crude oil futures prices and risk-neutral central moments of the price returns. I focus on several major oil-producing states but located in different geometric places over the U.S. – California, North Dakota, Texas, Oklahoma, and Pennsylvania, to analyze the effects of price moments on investment choices. The WTI crude oil is the most actively traded oil contract in the world. Its daily settlement prices and options can be collected over the past four decades. By using this unique dataset, I can observe the investment choices

on a monthly basis for about 600,000 individual oil wells from the time span January 2010 to September 2019 with the monthly-average of the daily prices and moments of oil price returns.

Investment under uncertainty is an ongoing debate in financial literature. The Real Options Valuation method has been used in many existing studies to explore the investment under uncertainty problem. In particular, the Real Options Valuation has a unique advantage in its power to explain the choices in investments considering sunk costs. Myers (1977) indicates that Real Options Valuation explains the “conservative” leverages of firms with market investment opportunities real options and whose value is negatively related to risky debt. Aguerrevere (2009) studies product market competition and finds that the relationship explains variations in firm returns among demand, concentration, and growth options. Aguerrevere (2009) suggests the importance of real options in influencing investment decisions under different market conditions. There are other studies on real options, including Décaire et al. (2020), who show the effect of peer exercises on real options, Berger et al. (1996), who explore the exit value of real options, and Mayers (1998) who analyzes convertible bond value with ROV. The Real Options Valuation suggests a negative relationship between price uncertainty and investment tendency – the higher price uncertainty leads to greater value of the “delay” option, leading to lower likelihood to invest. Other studies on the relationship between uncertainty and oil investment include Mohn and Misund (2009) in the aggregate level and Yang et al. (2008) in the micro and firm-level. The studies focusing on the impact of price uncertainty include Elder and Serletis (2010), Yoon and Ratti (2011), Ahmadi et al. (2019), Phan et al. (2019), Cao et al. (2020), Maghyereh and Abdoh (2020), and Doshi et al. (2018). However, some recent studies do not support the negative uncertainty-investment relationship. For instance, Miao and Wang (2007) show that uncertainty is positively associated with investment when the market is incomplete and risk cannot be perfectly hedged. Lambrecht (2017) provides a survey of the literature.

Kellogg (2011) and Covert (2015) examine learning and productivity in well drilling and fracking, respectively. Molls (2001) studies Oklahoma oil well production and finds that sunk costs and historic price change volatility help to explain oil well drilling decisions. Kellogg (2014) studies oil well drilling activity in Texas and concludes that oil well drilling activity is strongly affected by price uncertainty measured as price change volatility. He finds that greater volatility is associated with a larger probability of postponement of drilling activity. Using similar oil wells’ drilling data, Décaire et al. (2020) using data similar to Kellogg find that an information spillover effect influences drilling activity. Specifically, drilling in neighboring oil fields encourages oil well drilling in adjacent areas. Those authors however do not control for price uncertainty. Doshi et al. (2018) study quarterly investment expenditures by energy companies and also document an inverse relation between oil price volatility and aggregate quarterly expenditures among smaller firms. Boomhower (2019) and Muehlenbachs (2015) study firms’ decisions

to either abandon or environmentally remediate wells that are no longer productive. Anderson, Kellogg and Salant (2018) study production and drilling activity in Texas and find that while the expected price influences drilling activity it does not influence production activity. Chen and Linn (2017) study rig activity in the U.S. and find that rig activity in developed economies is positively related to oil futures prices.

Given that option values can be influenced by the skewness and excess kurtosis of the price change distribution of the underlying asset and that oil futures price changes exhibit such characteristics, I reexamine drilling activity in Texas as well as four additional major oil producing states accounting for both volatility as in Kellogg (2014), and the influence of peer activity as in Décaire et al. (2020), but importantly accounting for sunk costs as well as a comprehensive list of additional controls. The drilling choice does not involve a previous choice but does involve a sunk unrecoverable cost. On the other hand, the production choice, that is the selection between producing and temporarily shutting down a well, or permanently shutting down a well, do reflect a previous choice. The state of the well's production status coming into a time  $t$  will matter, do to both a cost of switching, as well as a change in per period operating cost. I explore these issues.

Optimal value-maximizing behavior conditioned on the expected price change distribution serves as a benchmark for assessing actual choices. If implicitly or explicitly, decision makers account for higher orders moments of the price change distribution, that is, they recognize the implications of these moments for the value of the real options they possess, then I should expect to see a relation between those moments and investment choices. This benchmark therefore serves as a null hypothesis and allows me to test whether that hypothesis is supported by the actual choice data. In order to set the stage, I begin by formulating the decision maker's problem and its solution conditional on the assumption of value maximization. I then translate that solution into an empirical model of choice that can be estimated with the data I have available. I focus on micro-level decisions at the level of the individual well and on the choices to drill, to produce, to temporarily shut down (mothball) and to permanently shut down. Its output prices are easy to measure with financialized crude oil commodities, oil futures and options, and oil well investment costs tractable with well depth and age. I use oil well production data of Pennsylvania, North Dakota, California, Oklahoma, and Texas and the BKM option-implied crude oil risk-neutral skewness and kurtosis to examine the impacts of changes in higher-order uncertainty on irreversible investments. By far, there is little research shedding light on the tail risks of price uncertainty on irreversible investment in the framework of real options. I present pieces of evidence that there is a strong association between skewness, kurtosis, and investment decisions, regardless of whether investors intend to apply the Real Options Valuation or not.

I set up several different empirical models to examine the impacts: the production status changing options, drilling options, and shutdown options in the oil wells' production history profile. I find significant effects of tail risks uncertainty on real options investment exercise in these models. I show that skewness and kurtosis have substantial power in explaining oil wells' production status and shutdown. The majority of the coefficients are statistically significant at a 1% significance level. When oil wells are not shut down and plugged in, I split producing status into two classes: "producing" and "mothballing." A more negative skewness (and a lower kurtosis) leads to more extraordinary expected "bad news" to increase the in-the-money "delay option" value, resulting in a greater tendency to postpone investments. A higher skewness (and lower kurtosis) results in lower expected "bad news" for in-the-money "delay" put option value, leading to a higher propensity to postpone "shut-in" plan. When focusing on the drilling and shutdown options, I run the regressions of the exercise dummy on skewness and kurtosis to find that the coefficients are significant and consistent with the Real Options Valuation predictions for shutdown options. The results suggest a research gap in the impacts of tail risks uncertainty on real options exercise (irreversible investments). Investors may implicitly apply the Real Options Valuation in making irreversible investments, considering the strong association of uncertainty on decision making.

Another significant contribution of this paper to related literature is the examination of the "Bad News Principle" on real options. If the "Bad News Principle" holds for real options investment, will its conclusion hold for higher-order uncertainty of price? I show that the "Bad News Principle" is valid for the higher-order central moments, as severer expected "bad news" (downside potential of price risk) correlates positively with the optimal exercise time and the probability of postponing investment. This finding formally addresses the asymmetry in the effect of price changes on real options' value. I build the relation between expected "bad news" and skewness and kurtosis. I reveal that the relationship between expected "bad news" and optimal timing reflects in the relationship between oil wells' production status and BKM implied skewness and kurtosis, so as the relationship between drilling and shutdown and the central moments.

#### **4. Model: The Choices to Invest, Produce, Temporarily Shut-in and Shut down**

##### **4.1 Option values and higher order moments**

Theoretically, higher-order expected moments of the price change distribution of the underlying asset can influence the value of options written on that asset. Jarrow and Rudd (1982), Corrado and Su (1996a, 1996b), and Brown and Robinson (2002) develop models of option valuation in which the log price change distribution of the underlying asset may exhibit higher order moments, that is skewness and kurtosis which deviate from the parameters of the Normal distribution. Jarrow and Rudd employ the Edgeworth expansion of an arbitrary probability distribution and Corrado and Su use the Gram-Charlier

Type A series expansion in their developments. As these methods essentially lead to equivalent results aside from the organization of terms, I will highlight only the Corrado and Su model.

Corrado and Su (1996a) develop the skewness- and kurtosis-adjusted Black-Scholes option pricing formula based upon the Gram-Charlier Type A series expansion. Subsequently, Brown and Robinson (2002) correct an error in the Corrado and Su model involving a sign in the expansion. The call option price when the log price change distribution of the underlying asset exhibits skewness and kurtosis equals (Brown and Robinson (2002)):

$$C = C_{BS} + \mu_3 \cdot Q_3 + (\mu_4 - 3) \cdot Q_4$$

where  $\mu_3$  and  $\mu_4$  are the skewness and kurtosis coefficients of the log price change distribution, respectively. The parameters  $Q_3$ , and  $Q_4$  are linear functions of the option's moneyness. If skewness equals 0 and kurtosis equals 3, and volatility is positive, the model reduces to the Black-Scholes price. Therefore, for a non-normal price change distribution, skewness and kurtosis in theory will influence an option's value. This result presents the motivation for considering that real investment choices, the exercise of real options may be influenced by not only expected future volatility but also expected future skewness and kurtosis.<sup>3</sup>

I do not directly explore the issue of the underlying process that may give rise to skewness and kurtosis. However, empirical simulations show that a general model that incorporates mean reversion and random jumps, in addition to Brownian motion, produces data that exhibit skewness and kurtosis for reasonable parameters. Such models have been found to fit oil futures price change data.<sup>4</sup>

As the choices I study involve decisions to exercise, or not exercise, an option, it is instructive to consider the interplay of skewness and kurtosis on the optimal exercise strategy within the context of the model described above. Figure I presents a graph in which skewness and kurtosis are allowed to vary along two axes and the trigger price at which it is optimal to exercise the option is identified on the third axis. The computations are based upon the following parameters: Current price = 92.00, constant annual risk-free rate of 2%, annual standard deviation in underlying security prices 20%, time to expiration one month, dividend yield 6%, and strike price 92.00. The figure shows that the optimal exercise price of the

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<sup>3</sup> See also Borland and Bouchaud (2004) and Potters et al. (1998). The option pricing model with non-zero skewness and excess kurtosis has been shown to perform better than the standard Black-Scholes option pricing model using market-observed option prices (Corrado and Su (1996b)).

<sup>4</sup> In the double exponential jump-diffusion process suggested by Kou (2002), the intensity of jumps on either side changes the higher-order distribution of prices. A greater intensity of jumps in the lower side decreases skewness and any non-zero jump intensity increases the excess kurtosis of distribution. A more recent study by Boyarchenko and Levendorskiĭ (2000) derives the theory for the optimal exercising rule under the Lévy processes of prices which they suggest could solve the real options problems with skewed prices and outliers, as well as Asmussen, Avram, and Pistorius (2004) who derive the put options with Lévy processes. See also Merton (1976), Bates (1997), Zhang (1997), Arnold and Crack (2000), Gukhal (2001), Lewis (2001), Kou and Wang (2004), Levendorskiĭ (2005), and Sepp (2008).

call option declines as skewness increases. The optimal exercise price does not experience a significant change as kurtosis increases.

The value of an option as described above under a non-Normal price change distribution serves as my base assumption for the valuation and exercise of real options when the underlying exhibits skewness and kurtosis. To establish the baseline framework for the evaluation of actual observed choices I next present a series of choice models based upon the assumption of value maximization in the presence of real options whose values should in theory be related to the skewness and excess kurtosis of the underlying price change distribution. Optimal choice behavior by producers should therefore be a function of those two parameters in addition to others including volatility. However, whether actual choices are related to skewness and kurtosis is an empirical matter which I evaluate using empirical choice models taking as the basis for their specifications the value maximizing theoretical frameworks.

#### **4.2 Models of Value Maximizing Choices**

Oil producers face several decisions which include the decision to drill and complete a well versus delaying investment, whether to abandon (shut-in) a well versus continuing to produce, and whether to mothball (temporarily shut-in) or continue to produce.<sup>5</sup> These choices represent options for the producer. Choices based upon a value maximizing criteria serve as the benchmark for the evaluation of the actual choices selected.

#### **4.3 The Production and Temporary Shut-in (Mothball) Choices**

I begin with a discussion of the choices regarding the producing status of a well, under the assumption that the status is chosen by selecting those decisions that are value maximizing at each future date over the expected life of the well. Assume the well has been drilled and completed and from that point on could either be producing or not producing (I will take up the decision to drill later). Assume a risk-neutral expected-payoff maximizing representative producer who must decide whether to make an irreversible investment when the producer's oil well's current status is "mothballing", i.e., production is temporarily shut-in. Define the state of a well as  $prod=p$  for "producing" and  $prod=m$  for "mothballing", respectively. The producer may immediately re-open the oil well and re-start production incurring an irreversible (sunk) investment cost, but will then receive the net payoffs from produced crude oil based upon a random future stream of output prices  $P$ , operating costs  $C^p$ , and production volumes  $Q$ . The cost to switch from mothballed to producing is assumed to equal  $\mathbb{C}^{m,p}$  and is not recoverable. However, the

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<sup>5</sup>I abstract from the decision to enter into a lease arrangement that then provides an opportunity to drill, taking the opportunity as a given.

producer may choose to defer the decision – use the “delay option” to delay the decision to a future date. By waiting the producer can observe an updated price level resolving uncertainty from  $t$  to  $t+1$ .

Similarly, the producer can choose to temporarily shut in an operating well, that is, to change to “mothballing” when current status is “producing”. The revenues from production (as discussed previously, a function of output prices  $P$ , operating costs  $C^p$ , and production volumes  $Q$ ) will be sacrificed and the representative producer maintain the oil well by costing maintenance cost  $C^m$ . The producer may choose to defer the decision with the “delay option” to postpone until next date. Price uncertainty is resolved by using the “delay option”. To summarize, I assume that if a well is producing an operating cost is incurred and if the well is mothballed a maintenance cost is incurred each period. The operating cost of the producing well is  $C^p$  and the maintaining cost of the mothballed well is  $C^m$ . The operating cost is incurred only when  $prod=p$ , and the maintaining cost is incurred each period only when  $prod=m$ . The value of the immediate choice to switch equals the discounted expected payoff stream less the switching cost  $\mathbb{C}^{p,m}$  (the net present value of the immediate choice). Assume that for a given well,  $C$ ,  $\mathbb{C}$ , and  $\theta$  are static parameters for each well where  $\theta$  includes observed and unobserved other variables affecting investment value. Assume that  $\mathbb{C} = \mathbb{C}^{m,p} = \mathbb{C}^{p,m}$ .

Define the value function  $V(P, Q, C, \theta, O(\mathbb{C}))$ , where value depends upon future oil prices  $P$  and the distribution of those prices, production quantity  $Q$ , operating cost  $C^p$ , maintaining cost  $C^m$ , other well characteristics  $\theta$ , and the options to “mothball” and to “produce” which are exercised at an investment cost  $\mathbb{C}$ . If the state of the well is mothballed then the option to switch to production is in force and if the state of the well is producing the option to switch to mothballing is in force. The options have values denoted as  $O^m(P, Q, C^m, \mathbb{C}, \theta)$  and  $O^p(P, Q, C^p, \mathbb{C}, \theta)$  which arise from the options to switch from “producing” to “mothballing” and from “mothballing” to “producing”, respectively. So the value function consists of three components: the profit that depends on  $P_t$  and  $Q_t$ , the costs  $C^p$  or  $C^m$ , and the real options  $O^m$  or  $O^p$ . The producer faces a multiperiod problem of selecting production as well as the optimal policy for exercising the options. The producer is assumed to follow a policy that maximizes an expected payoff function by choosing the optimal investment time  $t$  to exercise the option to switch, conditional on the current state of the well and an optimal policy concerning production.

Assume for illustration that the current state is  $prod = m$  (mothballing). The optimal choice to switch or continue with the current state of the well (that is delay switching) will define an optimal investment trigger price  $P^*$ . If the actual price exceeds the trigger price then the option would be exercised. That trigger price will be a function of the expected future distributional characteristics of the price change distribution. Define the implicit objective function  $\gamma$  where the optimal policy defines the selection of the state of the well at each date as:

$$\gamma = \max_{prod} V(P, Q, C, \mathbb{C}, \theta, O)$$

where  $P$  represents the expected price distribution at  $t$ ,  $Q$  is the set of quantities produced each period over the production horizon of the well,  $C$  represents the operating cost (*denoted*  $C^p$ ) stream of each period, including both fixed and variable cost if  $prod = p$ , and the cost of maintaining the well if  $prod = m$  (*denoted*  $C^m$ ).  $\mathbb{C}$  is the switching investment cost. The vector  $\theta$  contains both observed and unobserved well characteristics that affect production decisions, and  $O$  is the set of options the producer faces in each period (for instance,  $O = O^p$  if  $prod = m$ ). The behavior and distribution of  $P$  is assumed to be exogenously determined and is a function of time  $t$  and depends both on the expected price change, and the anticipated volatility, skewness, and kurtosis reflected in the price distribution.

$$P = P(\mu, \sigma, sk, k|t)$$

where  $\mu$  is the expected change,  $\sigma$  is volatility,  $sk$  is skewness, and  $k$  is the kurtosis of the price change distribution. Oil prices are determined in a world market and so this assumption conforms to both the actual market environment and the data (Kaminski (2012), U.S. Energy Information Administration <https://www.eia.gov/energyexplained/oil-and-petroleum-products/prices-and-outlook.php>).<sup>6</sup>

Define the multi-period dynamic problem of the producer as:

$$\gamma = \max_{prod} E_t(V(P, Q, C, \mathbb{C}, \theta, O)) = \max_{prod} \left( \pi(P_t, Q_t, C, \mathbb{C}, \theta | \mathbb{I}_t) + \frac{1}{(1+r)} E_t(V_{t+1} | prod_t) \right)$$

where  $\frac{1}{(1+r)}$  is the discount factor,  $\pi(P_t, Q_t, C, \mathbb{C}, \theta | \mathbb{I}_t)$  is the profit function at  $t$  given the current information set and  $E_t(V_{t+1} | prod_t)$  is the expectation of the continuing value function at  $t+1$  given the producing status choice at  $t$ ,  $prod_t$  assuming an optimal policy subsequent to  $t+1$ , where I have assumed that the expectation exists and that the price process is exogenous to the firm.<sup>7</sup> The value function at  $t$  can be rearranged to contain two maximized terms: the first term is the profit function at  $t$ , and the second is the discounted expected value function at  $t+1$  given producing status choice at  $t$  and an optimal policy following  $t$ . The production state  $prod$  is selected at  $t$  to maximize the sum of the two terms. The investment choice at  $t$  therefore depends upon the production state at  $t-1$ . If the representative oil producer chooses  $prod_t = p$  when  $prod_{t-1} = p$ , the above equation becomes:

$$\begin{aligned} V_t(prod_t = p, prod_{t-1} = p) &= \pi(P_t, Q_t, C, \mathbb{C}, \theta | \mathbb{I}_t) + \frac{1}{(1+r)} E_t(V_{t+1} | prod_t = p) \\ &= \tilde{\pi}(P_t, Q_t, \theta) - C^p + \frac{1}{(1+r)} E_t(V_{t+1} | prod_t = p) \end{aligned}$$

$$\tilde{\pi}(P_t, Q_t, \theta) - C^p = \pi(P_t, Q_t, C, \mathbb{C}, \theta | \mathbb{I}_t) \text{ for } prod_t = p \text{ when } prod_{t-1} = p.$$

<sup>6</sup> It is worth pointing out that a process for oil prices that exhibits mean reversion and random jumps as well as a continuous stochastic error, as has been found in the literature (Al-harthi (2007)) produces a price change distribution that exhibits both non-zero skewness and excess kurtosis for reasonable parameter values.

<sup>7</sup> The reader will recognize this as a Bellman equation. Molls (2001) presents a similar specification.



I henceforth drop the notation for the information set  $\mathbb{I}_t$  as it is implicit.

When  $prod_{t-1} = m$  and the representative producer chooses  $prod_t = p$ , the value function becomes:

$$V_t(prod_t = p, prod_{t-1} = m) = \tilde{\pi}(P_t, Q_t, \theta) - C^p - \mathbb{C} + \frac{1}{(1+r)} E_t(V_{t+1} | prod_t = p)$$

$\tilde{\pi}(P_t, Q_t, \theta) - C^p - \mathbb{C} = \pi(P_t, Q_t, C, \mathbb{C}, \theta | \mathbb{I}_t)$  for  $prod_t = p$  when  $prod_{t-1} = m$ . The only difference is that transitioning from “mothballing” to “producing” incurs a one-time sunk cost  $\mathbb{C}$ . Similarly if the representative producer chooses  $prod_t = m$  when  $prod_{t-1} = m$ , the value equation is:

$$V_t(prod_t = m, prod_{t-1} = m) = -C^m + \frac{1}{(1+r)} E_t(V_{t+1} | prod_t = m)$$

If the representative producer chooses  $prod_t = m$  when  $prod_{t-1} = p$ :

$$V_t(prod_t = m, prod_{t-1} = p) = -C^m - \mathbb{C} + \frac{1}{(1+r)} E_t(V_{t+1} | prod_t = m)$$

Where  $C^m$  is the maintenance cost in “mothballing” periods, and the only difference between the two equations is the term  $-\mathbb{C}$ , the one-time transitioning cost from “producing” to “mothballing”.

The well operator is assumed to select the choice that maximizes value conditional on the current state of the well ( $m$  or  $p$ ). Therefore, if maximizing behavior is assumed, the choice observed implies that choice has the largest current value, this of course also implies the choice carries with it an optimal policy going forward. Mothballing incurs a per period cost while a complete shut down does not, hence it would never be optimal to switch from complete shutdown to the mothball state.

When  $prod_{t-1} = m$ , the difference in the value equation between choosing  $prod_t = p$  and choosing  $prod_t = m$  will equal:

$$\begin{aligned} \Delta V_t(prod_{t-1} = m) &= \tilde{\pi}(P_t, Q_t, \theta) - C^p - \mathbb{C} + \frac{1}{(1+r)} E_t(V_{t+1} | prod_t = p) \\ &\quad - \left( -C^m + \frac{1}{(1+r)} E_t(V_{t+1} | prod_t = m) \right) \\ &= \tilde{\pi}(P_t, Q_t, \theta) - C^p - \mathbb{C} + C^m + \frac{1}{(1+r)} (E_t(V_{t+1} | prod_t = p) - E_t(V_{t+1} | prod_t = m)) \end{aligned}$$

When  $prod_{t-1} = p$ , the difference in the value equation between choosing  $prod_t = p$  and choosing  $prod_t = m$  equals:

$$\begin{aligned}
\Delta V_t(prod_{t-1} = p) &= \tilde{\pi}(P_t, Q_t, \theta) - C^p + \frac{1}{(1+r)} E_t(V_{t+1}|prod_t = p) \\
&\quad - \left( -C^m - \mathbb{C} + \frac{1}{(1+r)} E_t(V_{t+1}|prod_t = m) \right) \\
&= \tilde{\pi}(P_t, Q_t, \theta) - C^p + \mathbb{C} + C^m + \frac{1}{(1+r)} (E_t(V_{t+1}|prod_t = p) - E_t(V_{t+1}|prod_t = m))
\end{aligned}$$

The difference is twice the one-time transitioning cost from “mothballing” to “producing” (or from “producing” to “mothballing”),  $2\mathbb{C}$ .<sup>8</sup>

If there were no sunk costs associated with switching between production and mothballing then switching would be costless and instantaneous. In that case the future continuation value vanishes because the lagged state, that is the state coming into time  $t$ , will have no influence on future decisions. In the presence of sunk costs however, the lagged production state matters for the current choice of production state. If sunk switching costs are zero, the firm will produce if current profit is positive and will shut down if profit is negative. Mothballing would never occur since mothballing incurs a per period cost. Conversely the larger are the non-recoverable switching sunk costs, the more important is the lagged production status for the current period production choice.

The value effect from the optimal choice to switch from production to mothballing, or vice versa therefore implicitly defines a decision rule. The decision to switch will depend upon the distributional properties of future price changes and the remaining parameters of the model including the cost of switching. The profit function is determined by the oil price, quantity produced, and well characteristics,  $\tilde{\pi}(P_t, Q_t, \theta)$ . Given that the price is a random variable it can be characterized by the parameters of its probability distribution which I define as the expected future price, volatility, skewness, and kurtosis at  $t$ , conditional on the information set at time  $t$ . In addition, the difference in value function  $(-C^p + \mathbb{C} + C^m)$  is related to the operating cost, maintaining cost, as well as the transition cost. I re-write the difference in the value function  $\Delta V_t$  as reduced form equations:

$$\Delta V_t(prod_{t-1} = m) = \gamma_0^m + \gamma_P^m \cdot P_t + \gamma_Q^m \cdot Q_t + \gamma_C^m \cdot C + \gamma_\theta^m \cdot \theta + \gamma_{C^m}^m \cdot C^m + \gamma_{\mathbb{C}}^m \cdot \mathbb{C} + \varepsilon_t^m$$

and:

$$\Delta V_t(prod_{t-1} = p) = \gamma_0^p + \gamma_P^p \cdot P_t + \gamma_Q^p \cdot Q_t + \gamma_C^p \cdot C + \gamma_\theta^p \cdot \theta + \gamma_{C^m}^p \cdot C^m + \gamma_{\mathbb{C}}^p \cdot \mathbb{C} + \varepsilon_t^p$$

#### 4.3.1 The Empirical Model of the Choice to Produce or Mothball

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<sup>8</sup> I assume that the transitioning cost from “mothballing” to “producing” is the same as the cost from “producing” to “mothballing”, as a consequence, the difference between these two equations is twice the cost. If the transitioning costs are different ( $\mathbb{C}^{mp}$  and  $\mathbb{C}^{pm}$ ), the difference between these two equations become  $\mathbb{C}^{mp} + \mathbb{C}^{pm}$ .

I specify a dynamic panel binary choice model (i.e., panel probit) to examine the relationship between the producers' propensity to choose the “producing” status rather than the “mothballing” status and the characteristics (implied volatility, skewness, and kurtosis) of the oil price change distribution.

By estimating the above set of equations and testing whether coefficients on the lagged state dummy variables are significant and positive, I can test whether sunk costs matter in the presence of expected price uncertainty, but importantly test how expected prices and the expected future moments of the price change distribution influence actual choices. The above equations specify the problem of choosing producing status for each period  $t$  within the horizon 1 to  $T$ . Note that the implied value changes depend upon the lagged production status of the well. Combining the two expressions and introducing a lagged indicator for  $prod_{t-1}$  yields an integrated reduced form model. The binary choice problem depends upon the change in value from optimal selection of the production status choice going forward. Therefore, the implicit decision functions can be summarized as

$$I_{i,t} = \theta(\alpha + \beta \cdot prod_{t-1} + \lambda \cdot P_{i,t} + \omega \cdot Q_{i,t} + \gamma \cdot C_{i,t} + \beta \cdot \mathbb{C}_{i,t} + \tau \cdot \theta_{i,t} + \varepsilon_{i,t})$$

where  $P_{i,t}$  is a vector including the expected (output) price level and risk-neutral central moments (volatility, skewness, and kurtosis) and  $\lambda$  is a vector of coefficients.  $Q_{i,t}$  is the produced quantity of crude oil in month  $t$  for well  $i$ . Notice that the oil well production rate declines over time with well age.  $C_{i,t}$  contains all costs for  $prod = m, p$  including operating fixed and variable costs  $C_i^p$  and mothballing maintaining costs  $C_i^m$ . Well depth is positively correlated with the operating costs. As noted previously the profit function and the value function depend on the current producing status, so the price distribution moments (  $P_{i,t-3}^{18}, Vol_{i,t-3}^{18}, Skew_{i,t-3}^{18}, Kurt_{i,t-3}^{18}$  ) should interact with lagged producing status  $prod_{i,t-1}$ .  $\mathbb{C}_{i,t}$  is the investment cost to switch producing status ( $\mathbb{C}_{i,t} = C_{i,t}^{pm}$  for a producing well changing to mothballing and  $\mathbb{C}_{i,t} = C_{i,t}^{mp}$  for a mothballing well to re-open).  $\theta_{i,t}$  contains other observed determinants of well production choices, including drilling types, basin location, etc.

I define the following variables *horizontal drilling<sub>i</sub>*, *directional drilling<sub>i</sub>*, *basin<sub>i</sub>*, *productivity<sub>i</sub>*. “vertical drilling” is omitted so as to avoid the dummy variable trap. The error  $\varepsilon_{i,t}$  is assumed to capture any additional unobserved variables affecting oil well production decisions. The complete binary model can be estimated with the Maximum Likelihood Estimator. Here  $\Phi$  is the standard normal distribution function for the probit model.

$$\begin{aligned}
prod_{i,t} = & \Phi(\alpha_0 + \beta_0 \cdot prod_{i,t-1} + \beta_1 \cdot P_{i,t}^{18} + \beta_2 \cdot \{prod_{i,t-1} = 1\} \cdot Vol_{i,t}^{18} \\
& + \beta_3 \cdot \{prod_{i,t-1} = 0\} \cdot Vol_{i,t}^{18} + \beta_4 \cdot \{prod_{i,t-1} = 1\} \cdot Skew_{i,t}^{18} \\
& + \beta_5 \cdot \{prod_{i,t-1} = 0\} \cdot Skew_{i,t}^{18} + \beta_6 \cdot \{prod_{i,t-1} = 1\} \cdot Kurt_{i,t}^{18} \\
& + \beta_7 \cdot \{prod_{i,t-1} = 0\} \cdot Kurt_{i,t}^{18} + \nabla_1 \cdot \overline{basin}_i + \delta_1 \cdot horizontal\ drilling_i \\
& + \delta_2 \cdot directional\ drilling_i + \delta_3 \cdot undetermined\ drilling_i + \gamma_1 \cdot age_{i,t} \\
& + \gamma_2 \cdot depth_i + \gamma_3 \cdot productivity_i + \varepsilon_{i,t})
\end{aligned} \tag{1}$$

#### 4.4 The Choice to Drill Now or Delay

Similarly, new drillings are real options for oil producers. Kellogg (2014) shows that the drilling choice is positively related to price but negatively to price volatility. Drilling are the call options to producers. I predict that skewness and kurtosis are significant influences for the real options decisions according to the “Bad News Principle” and I use the following econometric model to examine the impacts of moments:

Similar to the exercise of the “producing” or “mothballing” option, producers will exercise the “drilling” option if the “waiting option” value is less than the immediate exercise value of drilling. The choice problem is one in which drilling has not occurred but can at a fixed cost, generating a stream of net cash flows in the future. Conversely, the choice to not drill at date  $t$  but to wait, will be governed by the value of waiting with the option to exercise (invest) at a future date. In summary form the choice is to select to drill to maximize a value function that includes the option to drill

$$\gamma = \max_{drill} V(P, Q, C, \mathbb{C}, \theta, O^d)$$

where  $P = P(\mu, \sigma, sk, k)$  and  $O^d$  is the drilling option. Conversely, the choice not to drill implies that the value of the option to delay exceeds the immediate value of drilling.

##### 4.4.1 The Empirical Model of the Choice to Drill or Delay

The empirical reduced form model is specified as, where  $h$  is the decision function

$$I_{i,t}^{drilling} = \theta(\alpha + \beta \cdot prod_{i,t-1} + \lambda \cdot P_{i,t} + \omega \cdot Q_{i,t} + \gamma \cdot C_{i,t} + \beta \cdot \mathbb{C}_{i,t}^{drilling} + \tau \cdot \theta_{i,t} + \varepsilon_{i,t})$$

Unlike the choice to continue production or mothball which is a dynamic decision-making process, the exercise of the drilling option is a one-time event. The purpose of the Cox Proportional Hazard model is to evaluate simultaneously the effect of several factors on survival. In the case at hand the initial state is that the well has not been drilled. The well survives in this state until drilling begins. In other words, it allows me to examine how specified factors influence the rate of a particular event

happening (e.g., drill) at a particular point in time. The Cox proportional hazard model asserts that the rate of an event occurring at a point in time is related to a set of explanatory variables and a baseline rate. Within the context evaluated here, the model allows me to examine how specified economic factors or characteristics influence the rate of a particular event happening at a particular point in time. The rate is commonly referred to as the hazard rate. The Cox proportional hazard model considers that the probability of exercising the option (the event occurs) increases as time passes; it is rather than a static process in which the likelihood to exercise an option does not change over time. For example, the probability that a new oil well will be drilled in 10-th month is higher than that in the second month after the drilling option becomes available. The Cox model will fit this element when estimating the coefficients. So, the Cox proportional hazard model is more suitable for exploring drilling options' exercises.

Define the variable  $exer_{i,t}$  for well  $i$  at date  $t$ . The variable  $exer_{i,t}$  which takes the value 1 if the choice to drill at  $t$  occurs and 0 otherwise. Specifically, drilling is defined as those months which occur on and after the spud dates of oil wells, and the dates on and before well completion. Once drilling has begun, the costs incurred are not reversible. Hence the choice to drill is an irreversible investment decision. That is, once drilling begins the cost is sunk. As emphasized earlier, the model allows a test of several hypotheses. First, that expected prices, volatility, skewness and kurtosis, influence the choice to drill. Second, a test of the hypothesis that sunk costs matter. The estimated model is specified to include the variables  $P_{i,t-3}^{18}$ ,  $Vol_{i,t-3}^{18}$ ,  $Skew_{i,t-3}^{18}$ , and  $Kurt_{i,t-3}^{18}$  as defined earlier and in Appendix A. I define dummy variables for the primary drilling operations: horizontal drilling, directional drill and undetermined, with vertical drilling excluded from the set of dummies to avoid the dummy variable trap. The model also includes two variables that reflect costs  $depth_i$  is the well depth in tens of thousands of feet  $t$ , and  $total\ costs_i$  is equals the sum of drilling, tie-in, and completion costs. I include completion costs as these would be the costs required to implement actual production. Expected productivity,  $productivity_i$ , is a measure of expected reserves based upon first test oil volume in bbl. The model is expressed as the following where  $h_{0,i}$  is the hazard rate when all explanatory variables equal 0. The hazard rate represents the probability of drilling, given that no drilling has occurred up to a specific time.

$$\begin{aligned}
h_i(t) = & h_{0,i}(t) \exp(\beta_1 \cdot P_{i,t-3}^{18} + \beta_2 \cdot Vol_{i,t-3}^{18} + \beta_3 \cdot Skew_{i,t-3}^{18} + \beta_4 \cdot Kurt_{i,t-3}^{18} \\
& + \nabla_1 \cdot \overline{basin}_i + \delta_1 \cdot horizontal\ drilling_i + \delta_2 \cdot directional\ drilling_i \\
& + \delta_3 \cdot undetermined\ drilling_i + \gamma_1 \cdot depth_i + \gamma_2 \cdot productivity_i \\
& + \gamma_3 \cdot total\ costs_i)
\end{aligned} \tag{2}$$

#### 4.5 The Choice to Permanently Shut Down or Continue

Shutting down an oil well is a put option to producers. I derive the payoff functions of the producing and shutdown statuses in the following equations: The producer will choose “shutdown” over “producing” when  $-\mathbb{C}^{shutdown}(\theta) > V_t(prod_t = p)$ <sup>9</sup>. Where  $\mathbb{C}^{shutdown}$  is the one-time sunk costs to switch from the producing status to shut down, which include but are not limited to the costs of the well hole plug-in cost, the cost to dispose of well pollutes, and transportation cost. Also, I have  $P = P(\mu, \sigma, sk, k)$ . Intuitively, it is optimal to switch to shutdown status from producing if  $-\mathbb{C}^{ps} > V_t(prod_t = p)$  (Notice that  $V_t(shutdown) = 0$ ). When maintaining producing status it is not economical for oil wells, i.e., when  $V_t(prod_t = p)$  becomes negative as operating costs surge or price declines, closing a well may be optimal. Similar to the derivation for the “defer” option, I can derive the difference in value functions (when  $prod = p$  vs. when  $prod = s$ ):

$$\begin{aligned}\Delta V_t(prod_{t-1} = p) &= \tilde{\pi}(P_t, Q_t, \theta) - C^p - \mathbb{C} + \frac{1}{(1+r)} E_t(V_{t+1}|prod_t = p) - \frac{1}{(1+r)} E_t(V_{t+1}|prod_t = s) \\ &= \tilde{\pi}(P_t, Q_t, \theta) - C^p - \mathbb{C} + \frac{1}{(1+r)} (E_t(V_{t+1}|prod_t = p) - E_t(V_{t+1}|prod_t = s))\end{aligned}$$

and:

$$\begin{aligned}\Delta V_t(prod_{t-1} = s) &= \frac{1}{(1+r)} E_t(V_{t+1}|prod_t = s) - \frac{1}{(1+r)} E_t(V_{t+1}|prod_t = s) \\ &= \frac{1}{(1+r)} (E_t(V_{t+1}|prod_t = s) - E_t(V_{t+1}|prod_t = s))\end{aligned}$$

I use a similar model to examine the relationship between skewness (kurtosis) and shutdown real option exercise:

$$I_{i,t}^{shutdown} = \theta(\alpha + \beta \cdot prod_{t-1} + \lambda \cdot P_{i,t} + \omega \cdot Q_{i,t} + \gamma \cdot C_{i,t} + \beta \cdot \mathbb{C}_{i,t}^{shutdown} + \tau \cdot \theta_{i,t} + \varepsilon_{i,t})$$

$$\begin{aligned}exer_{i,t} &= h(\alpha_0 + \beta_1 \cdot P_{i,t-3}^{18} + \beta_2 \cdot Vol_{i,t-3}^{18} + \beta_3 \cdot Skew_{i,t-3}^{18} + \beta_4 \cdot Kurt_{i,t-3}^{18} + \nabla_1 \cdot \overline{basin}_i \\ &\quad + \delta_1 \cdot horizontal\ drilling_i + \delta_2 \cdot directional\ drilling_i + \delta_3 \cdot undetermined\ drilling_i \\ &\quad + \gamma_1 \cdot depth_i + \gamma_2 \cdot productivity_i + \gamma_3 \cdot total\ costs_i + \varepsilon_{i,t})\end{aligned}$$

It is optimal to exercise the shutdown real option when the output price falls below the investment threshold, which is determined by the relative size of the “defer” to shut down option and the immediate shutdown value.

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<sup>9</sup> I ignore the case when a shutdown well is re-opened as this is a very rare case due to significant costs. I also ignore the case when switching from mothballing to shutdown as the data do not distinguish this case from direct shutdown.

Exits of producers are also real options. By exercising the option to shut down, producers avoid the costs associated with operating the properties when continuing production is no longer economical. For instance, declines in oil price and rise in operating costs. Dixit and Pindyck (1994) show discuss the option to abandon, whose threshold decreases with the upside potential of price. Moel and Tufano (2002) find that the propensity to exercise exit options for gold mines is negatively associated with uncertainty after controlling for price. The “Bad News Principle” applies to the “mothball” option and exit option. I predict the following: the probability of shutting down a well falls when expected good news becomes a more likely event; wells are more prone to be closed when upside potential price changes are smaller.

#### 4.5.1 The Empirical Model of the Choice to Shut Down or Continue

The shutdown model is similar to the drilling model but with the dependent variable as the dummy “exer”=1 for well closing months, i.e., the month when the oil well was closed. The initial state is therefore that the well is producing. The choice variable “exer” equals 1 for the closing month and equals 0 for those months before the closing month. The explanatory variables are the same as those in the drilling model in except for  $productivity_i$ , which is replaced with a variable labeled “last12” which equals the production rate of the well for the last 12 months’. A more productive producing oil well (higher “last12”) is less likely to be closed. The structure of the empirical model is:

$$\begin{aligned}
 h_i(t) = & h_{0,i}(t) \exp(\beta_1 \cdot P_{i,t-3}^{18} + \beta_2 \cdot Vol_{i,t-3}^{18} + \beta_3 \cdot Skew_{i,t-3}^{18} + \beta_4 \cdot Kurt_{i,t-3}^{18} \\
 & + \nabla_1 \cdot \overline{basin}_i + \delta_1 \cdot horizontal\ drilling_i + \delta_2 \cdot directional\ drilling_i \\
 & + \delta_3 \cdot undetermined\ drilling_i + \gamma_1 \cdot depth_i + \gamma_2 \cdot productivity_i \\
 & + \gamma_3 \cdot total\ costs_i)
 \end{aligned} \tag{3}$$

## 4.6 Predictions

Based upon the generic model described in section 4.1 in which call option values are a function of both volatility, skewness and kurtosis and the developments above on the optimal choice problems, Table I presents the predicted relations between the probability of each choice studied and expected volatility, skewness, kurtosis and prices. In addition, the table lists well characteristics discussed in the following section, and the predicted influence of those characteristics on the choices studied.

## 5. Data and Methods

### 5.1 Oil Well Lifecycle - Drilling, Producing, Mothballing, and Shutdown

The lifecycle of an oil well goes through several stages, from testing to plugging in. Firstly, a seismic survey of an oil well usually includes testing for the estimated quantity of expected total production and measured depth of the oil reserve. It takes about three months on average to complete the drilling process. The porous rocks are broken by a rotary drilling rig, which is usually rented and thus incurs rig-rental cost. The method also involves removing the mud from the drilling - a transportation cost. The drilling also uses materials to stabilize the rock with cement and steel - a casing process that incurs completion costs. Water may be pumped into the reservoir to increase pressure and help pump out the oil. While the well is producing, additional horizontal wells (infill wells) may be drilled to increase oil production from the same reservoir – these are the infill wells.<sup>10</sup>

After drilling ends and the well is completed, production may begin, and the oil well moves to its second stage, “producing.” At this stage, producers earn revenues from produced crude oil and which is positively related to the produced quantity of oil product and market oil prices, as well as is negatively related to the operating costs, including the administration cost and equipment depreciation, thus net revenue from operating a producing oil well comes from two parts – the generated income from selling the produced crude oil at market prices and the cost to maintain the production. At this stage, oil producers consider the expected oil price (and distribution moments) in the futures and how much the cost to maintain the production to decide on production status.

Mothballing is a temporary suspension of production. When facing bad news for production, such as price shocks or a surge in operating costs, producers have the option to mothball the producing oil well. The difference between mothballing and shutdown (plugging in) is that a mothballed well can be quickly re-opened to resume production, and the cost of doing so is lower than reopening a plugged well, where in fact it might be impossible to reopen a plugged well and a plugged in well is permanently unusable. Mothballing an oil well incurs less cost than completely closing a well and re-opening the mothballed well costs less than drilling a new one. Mothballing status is an intermediate status between “producing” and “shut down” but it requires a medium level of maintaining cost to keep this status. The benefit of maintaining the “mothballing” status is that oil producers still have the option to resume production.

The last stage of an oil well is “shutdown” – when producing or maintaining the well is no longer economical or when the oil wells are exhausted. As I discussed before, closing an oil well takes far more cost than mothballing a well – it takes plug materials to permanently close a well to prevent pollution or expansion of the abandoned wells. Most used plug material is cement. Re-opening a plugged oil well is generally not possible and where possible is extremely costly.

## **5.2 Oil Well Investment and Production Data**

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<sup>10</sup> For a detailed description of the drilling process see <https://production-technology.org/>.



### 5.2.1 Records of Oil Wells' Drilling, Production, and Closing Dates

Oil well production rates and dates and dates production begins and the dates production ends are obtained from Enverus.<sup>11</sup> Each state of the United States typically requires producers to file well drilling and production records with a state agency on a monthly basis. Enverus gathers these data records and assembles them into a common format data base. Data obtained include a well identifier code (an unique well identification 14-digit number: American Petroleum Institute, short for "API"), the well's operator company's names, the well's monthly oil production rates, gas production rates, and water production rates, well's measured depth in tens of thousands of feet, a well lease identifier (and lease names), the basin where the well is located, well's field name, and location's county and state names, total drilling costs, the date drilling begins (the spud dates), the dates when the well begins production (the completion dates), well's peak production rates, well's first 6-months' and first 12-months' production rates, and age of the well measured as the number of months from the first production date, and the date a well is closed off so that it stops producing (the "last production" dates).

Records on over 600,000 oil wells are obtained for the states of Texas, North Dakota, Oklahoma, California, and Pennsylvania. The total number of well-month observations equal roughly 53,000,000. According to their monthly oil production rates and their first and last production months, I construct the production histories/status for each of the 600,000 oil well's on a monthly basis. The earliest date is January 2010 and the last date is September 2019. The spud date identifies when drilling begins and investment costs are incurred. The production status histories, allow recovery of the investment decisions (choices between producing and mothballing) of oil producers through every oil well's entire production history during the ten-year period studied. The histories allow identification of the investment date (which I classify as the spud date) and the decisions to temporarily suspend production and to abandon (shut in) a well. The production histories therefore provide information on the state of the well at any date. Each oil wells' drilling and shutdown profiles are constructed from the well spud dates and their last production dates.

### 5.2.2 Data on Expected Revenues and Costs

*Age, Depth, and Productivity:* The likelihood of resuming, mothballing, drilling or closing a well depends on potential productivity. Each oil well's total reserve is used to proxy for expected productivity. The accumulated oil production rate is used as the proxy for operating and mothballing oil wells; oil wells' last 12-months' production rate is used as the proxy for the productivity of oil wells facing closing down decisions. Age controls for the decline in production rates over time as oil reserves are exhausted. An oil

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<sup>11</sup> Enverus.com. I am grateful to Enverus for providing me with access to their well production and cost datasets and especially to Ms. Annie Shen and Mr. Jason Eleson for their very generous help.

well's oil production rates changes over time and can be predicted by the Hubbert curve with the prediction function  $Q(t) = \frac{Q_{max}}{1+e^{a(b-t)}}$ <sup>12</sup> which decreases with age (the  $t$  in the function). Well age, depth, reserve, last 12-months' production rate, and cumulative oil production rate are rescaled to fit the magnitudes of other variables in my model and are measured as natural logs. Age equals the log of the number of months since production began. Well depth (in natural logs of tens of thousands of feet) is included to control for differential operating costs as those costs vary with depth.<sup>13</sup>

*Drilling and Shutdown Costs:* With the drilling cost dataset from Enverus, I can measure individual wells' total drilling costs (total drilling costs consist of the drilling, completion, and tie-in costs), which are positively related to oil wells' closing costs.<sup>14</sup> So, I use the drilling costs data to measure the total costs to drill a new oil well in a lease field and as a proxy for the cost to plug in a closed oil well, however plugging an existing well incur on average less costs than drilling a new well.<sup>1516</sup> The drilling costs are deflated<sup>17</sup> in order to control for the difference in dollar value across time. Natural logs of the data are used to fit to the scales of other variables.

*Drilling Types and Basins:* Directional or horizontal drilling allows producers more flexibility and precision in reaching and extracting oil/gas compared to vertical drilling and directional drilling generally yields greater oil extraction. By some estimates however vertical drilling costs only about 25-50% of what it costs for a horizontally drilled well.<sup>18</sup> Basin is a depression, or dip, which may be filled with oil. Geographic characteristics of the basin in which the well is located can influence the amount of oil produced. I control for well head location by identifying the basin in which the well is located. Controlling for the basin location accounts for differential geographic issues such as crude oil quality, gravity<sup>19</sup>, sulfur, and rock porosity.

### 5.2.3. Construction of the Dataset

<sup>12</sup> See refence: Claerbout, J., & Muir, F. (2008). Hubbert math. *Sepstanford. edu*.

<sup>13</sup> OilGasEquity.com. (n.d.). *Drilling & Completion Facts*. OilGasEquity.com. Retrieved September 6, 2022, from <https://www.oilgasequity.com/resources/drilling-completion-facts/#:~:text=The%20study%20found%20that%20drilling,%241.6m%20for%20drilling%20alone>.

<sup>14</sup> This is confirmed by a conversation with industry professional.

<sup>15</sup> Raimi, D., Krupnick, A. J., Shah, J. S., & Thompson, A. (2021). Decommissioning orphaned and abandoned oil and gas wells: New estimates and cost drivers. *Environmental Science & Technology*, 55(15), 10224-10230.

<sup>16</sup> HydroCarbonWell.com (n.d.). *How Much Does It Cost to Plug an Oil and Gas Well?* Hydrocarbon Well Services. Retrieved September 6, 2022, from <https://hydrocarbonwell.com/how-much-does-it-cost-to-plug-an-oil-and-gas-well/>

<sup>17</sup> Both price and costs are deflated by inflation rate, source of CPI: <https://fred.stlouisfed.org/>.

<sup>18</sup> Rig Worker. (2022, September 5). *Horizontal well costs - oil well drilling*. Rig Worker. Retrieved September 6, 2022, from <https://www.rigworker.com/oil-well/horizontal-well-costs.html>

<sup>19</sup> API gravity, is a measure of how heavy or light a petroleum liquid is compared to water. High API gravity referred to as 'light'. Oil with a high sulfur content is considered "sour" and low-sulfur oil is "sweet". Light-sweet oil typically commands a higher price.

I assemble the following data for each well in the set of wells with spud dates between March 2010 and September 2019. I access the age (in months) and depth (in feet) of oil wells, oil well's accumulated production rates (monthly in bbl), total reserve (in bbl), drilling types (horizontal, directional<sup>20</sup>, vertical, or undetermined), oil basin (for instance, Fort Worth, Palo Duro, and east Texas coastal), last 12-month's production rates (in bbl), total drilling costs (including drilling cost, completion cost, and tie-in cost) (in millions of USD), oil lease names, and state in which the well is located. I examine activity in Texas, North Dakota, Oklahoma, California, and Pennsylvania, the five states with the most complete records as well as large numbers of wells. I use these variables to control investment costs, operating costs, expected oil production rate, expected lifespan of the oil wells, etc. I define total drilling costs to include both completion cost as well as tie-in cost (the cost of tying into a pipeline for transportation) as these costs must be incurred before oil can physically be produced and transported for sale.

I exclude observations with any missing variables. Table III presents descriptive statistics for the sample production data.<sup>21</sup> This table summarizes the variables in the production dataset (except for the *exer(drilling)* and *exer(shutdown)* variables which are only available in the drilling dataset and the shutdown dataset, respectively). On average, half of the well-month observations are from operating oil wells – they are producing crude oil, but the other half of the observations are from mothballed wells. This mean in “*prod*” suggests that mothballed oil wells are not rare cases – actually, a shut-in oil well is very common. Notice that the variables *age*, *cum. oil production*, *reserve*, *depth*, and *last12* are taken natural logarithm to fit to the scale of other variables. So, on average, the oil wells in my dataset is about 126 months' old – they have been producing for about 10 years. This is consistent with the fact that most oil wells can produce up to about 20 years. On average, a decision to drill an infill well happens in the middle of the oil lease's lifespan – the mean of *exer(drilling)* is 0.3281. It suggests that from the drilling of the first oil well in a lease, it takes about half of the total production life of a lease to drill an infill well to boost production. *Exer(shutdown)* is much smaller than *exer(drilling)*. Notice that a shutdown oil well will not remain in the dataset for a long time. Among all the oil wells the majority are vertical drilling oil wells. Horizontal and directional drilling wells consist of about less than 20% of the sample. Texas occupies a large proportion of oil wells in the sample – about 58.33% are Texas oil wells. North Dakota only consists of a small portion of the sample (2.34%).

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<sup>20</sup> In the Enverus well dataset, directional drilling is distinguished from horizontal drilling; but (horizontal) directional drilling is defined as a specific type of well drilling method and generally belongs to the directional drilling class. Horizontal drilling is different from directional drilling in that it is generally used to boost production rates of an existing reservoir by drilling several additional wells along the field.

<sup>21</sup> Sample sizes used in estimation may differ due to missing observations for some explanatory variables.

*Drilling:* In my examination models of oil drilling decisions, I used only the first “infill” wells within a lease – that is, the first wells being drilled after the spuds of the first oil wells within a lease. Using infill well drilling decisions represents a clean way of defining optionable choices to drill as they are made only after the initial stage of the drilling on the leased area. Kellogg (2014) and Décaire et al. (2020) follow a similar approach in their analyses. The “infill” wells are the oil wells drilled to increase production rates of existing oil well fields when there already exists a producing oil well in the leased field.

*Shutdown and Production:* In my examination of well production status changes and shutdown choices, I used all oil wells ever producing within the months March 2010 – September 2019, which is the complete sample of oil wells from Enverus in the states California, Oklahoma, North Dakota, Texas, and Pennsylvania. Instead of using just the first infill wells as in the oil well drilling sample, I used all producing wells from 2010 to 2019 as all producing oil wells become a put option for oil producers – they could choose to shut in the oil wells to reduce operating cost but maintain the option to re-open it. In the case of extremely low expected oil prices and price change uncertainty, they might permanently close the existing oil wells to reduce potential financial loss, since a shut-in oil well still requires a maintaining cost to enable future re-opening but a closed oil well does not incur further costs but a one-time plugging in cost.

As mentioned previously, drilling an oil well is an example of exercising an irreversible investment, and closing an oil well of abandoning a previous investment. I recover the beginning dates of each oil well’s investment horizon from their oil lease information. It is the spud month of the first oil well in production from the same oil lease field when drilling an additional oil well becomes an exercisable option within the lease.<sup>22</sup> The option remains exercisable until the last oil well in the lease ceases to produce. I recover the closing dates of each oil well from their last production dates on record. The beginning of the oil well’s potential disinvestment horizon is the oil well’s first production date. To summarize, I take the oil well lease after the spuds of first wells as the real options to drill (thus call options to open a new oil well) until the spuds of the first infill wells which are the dates options are closed, and the producing oil wells as the put real options to shut in or close wells; in addition, a mothballed and temporarily shut-in oil well is a call option to re-open productions.

### **5.3 Oil Futures Prices and Implied Moments**

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<sup>22</sup> Also see definitions of “infill wells” in this section. This “infill” wells follow the set up in Kellogg (2014) and Décaire et al. (2020).

I use BKM's (Bakshi, Kapadia, and Madan (2003)) risk-neutral model-free option-implied central moments to estimate volatility, skewness, and kurtosis of the oil price change distribution. These central moments are constructed from futures and futures' options prices.

Daily crude oil (symbol: CL) futures prices and options on futures prices (symbol: LO) are obtained from the Chicago Mercantile Exchange, CME, Group Datasets (End of Day Complete database). The data are used to compute a proxy for the expected oil price and option-implied volatility, skewness, and kurtosis at future time horizons. Only option prices with maturities between 10 and 180 days, the most liquid options contracts, are utilized, as contracts expiring or far from maturity are traded thinly and whose settlements are more noisy. Options and futures prices that are clearly out of line relative to averages are assumed to be recording errors and are excluded. Risk-free rates are obtained from the OptionMetrics files through WRDS. Risk-free rates are extrapolated and interpolated to ensure sufficient data for maturities spanning 10-180 days.

I follow Chang, Christoffersen, and Jacobs (2013) and Ruan and Zhang (2018) in computing the risk-neutral central moments of the price change distribution. First the data are filtered by a) dropping option prices lower than  $3/8$  (minimum tick), b) dropping deep-in-the-money options (put options with an exercise price higher than 103% of futures price and call options with an exercise price lower than 97% of futures price), c) dropping prices on days with fewer than 2 put or call prices, and/or options prices violating arbitrage conditions.<sup>23</sup> Second, to expand the option prices set from the CME option settlement prices, I expand the range of moneyness within a date and maturity using existing observations' moneyness values. The expansion uses the cubic spline interpolation method. I expand moneyness for each date and maturity to the range between 0.0001 and 3 and use the implied volatility (from CME's modified Black-Scholes option pricing model) to interpolate and extrapolate implied volatilities in that range. Third, I use the implied volatilities to infer option prices using the Black-Scholes option on futures pricing model to obtain a smooth option price function in the moneyness 0.0001 and 3 range for each date and maturity. Fourth, futures prices and in the moneynesses [0.0001,3] option prices are then used to calculate option implied volatility, skewness, and kurtosis following the algorithm proposed by Bakshi, Kapadia, and Madan (2003) (BKM). The trapezoidal integration method utilized by Chang, Christoffersen, and Jacobs (2013) and Ruan and Zhang (2018) is employed in the calculations.<sup>24</sup> Fifth, I compute 18-month BKM risk-neutral central moments. Kellogg (2014) points out that "18 months" is a typical horizon for oil operators to observe prices. Décaire et al. (2020) use similar measures for price and volatility. Also, Slade (2001) mentions that "Most firms use a long-run commodity price" for decision

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<sup>23</sup> Arbitrage conditions: excluding observations of option prices when a call option's price is greater than or equal to present value of futures prices or when a call options' price is lower than or equal to the present value of futures prices minus present value of strike prices.

<sup>24</sup> For more detail on the trapezoidal integration of central moments, please see p.587-588 of Ruan and Zhang(2018).

purposes. Those producers usually consult forecasted long-run output prices for investment decisions according to survey results with managers of copper producers. Specifically using futures prices I estimate the term structure of oil return realized central moments, and then compute 10-180 days central moments to interpolate the one-month maturity central moments.<sup>25</sup> The one-month central moments and term structure are then used to extrapolate 18-month maturity central moments. However, for the production examination model, I use the one-month price and central moments, as transitioning between producing and mothballing can happen immediately.

I assume the physical timing to drill or shutdown is not immediate but the time between the formal evaluation and actual implementation of the choice occurs with a lag.<sup>26</sup> Three-month lags of the estimated central moments are employed in the analyses presented later for the drilling and shutdown examination models.<sup>27</sup> 18-month forward central moments are calculate by taking averages of the 18-month maturity futures settlement prices. Both the 18-month and the one-month oil price are deflated and scaled by 100 to fit to the scale of the risk-neutral moments. The oil producers can observe monthly average forward price and distribution moments to determine if to continue producing or shut in it. My sample is on a monthly-basis for each individual well and I can only observe the changes in oil well producing states in the months.

## **6. Empirical Results**

### **6.1 Higher Order Central Moments and Oil Production Choices**

This sub-section explores the impacts of price distribution moments (volatility, skewness, and kurtosis) on irreversible investment decisions. I explore the problem of the exercise of the delay option, that is the choice to continue production versus temporarily shutting down, after controlling for a complete set of controls, including oil well productivity, existing producing status, and drilling type. I examine the impacts of the distribution moments in two steps. Firstly, in section 6.2, I look at the effect of volatility on oil production investment decisions, i.e., the relationship between volatility and the probability of choosing “producing” status. The results show that increases in expected future price volatility are inversely related to increased real options value, and the increase in real options value postpones irreversible investments. My conclusions are consistent with the existing literature, increases in price volatility lead to a higher probability of delayed investment as the delay option value increases. Secondly, in section 6.3, I examine the impacts of the higher-order moments on oil production investment decisions. I examine the relationship between skewness and kurtosis and the probability of choosing “producing” status to explore the effects of the non-normality of the price change distribution on the

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<sup>25</sup> The method follows Kellogg (2014, Appendix A).

<sup>26</sup> This follows Kellogg (2014) – it usually takes three months to commence drilling after decision.

<sup>27</sup> Computation details are available upon request.

option to delay and the likelihood of investing immediately. In this part, I explore the relation between the higher-order moments of the price change distribution on the choice to delay decisions. Based upon the modified option pricing model in which skewness and kurtosis impact option value. I expect to find that skewness and kurtosis affect the choice to produce or mothball. More positive skewness should be associated with a greater probability of mothballing an operating oil well. As for kurtosis, if resuming production (and mothballing a well) is a not-deep-in-the-money option, I expect to find a positive relationship between kurtosis and the likelihood of re-opening a mothballed well (to shut in a producing well).

## 6.2 The Choice to Produce or Mothball: Volatility and Sunk Costs

More significant uncertainty (increases in price volatility) increases the severity of “bad news” and in theory leads to a postponement of investment. As price volatility increases, the delay option value increases for call and put options. When the delay option value exceeds the value of “immediate investment” producers are more likely to choose to postpone investment (choosing the delay option) rather than invest immediately (choosing the immediate investment). This choice leads to the relationship between price volatility and the probability of changing the producing status of a wells. Changing from “mothballing” to “producing” is an investment (which incurs costs and brings cash flows from selling crude oil from production). If the current state of a well is ‘mothballed’ to resume production requires a sunk cost investment. That is the value of the choice depends upon the state of the well at the time of the decision. The predicted coefficient for the interaction of “mothballing” lagged status and volatility is negative, higher uncertainty reduces the probability of making the investment to switch immediately when the current state is mothballed as the delay option value exceeds the value of resuming production.

Similarly, the interaction of “producing” lagged status and volatility is positive, as more significant uncertainty reduces the probability of exercising the option to “temporarily shut-in” the well. The value of remaining open and producing exceeds the value of shutting in the well. The dependent variable  $prod=1$  for “producing” indicates that the oil well’s status is unchanged when oil price uncertainty increases. I illustrate the predicted coefficients for the interactions for oil price volatility in section 4 in detail.

Column (1) of Table IV, reports panel probit estimation results in which the choice variable is defined as follows. The dependent variable here is the oil well’s production status, and  $prod = 1$  for “producing” and  $prod = 0$  for “mothballing.” The production choice  $prod=1$  if production is chosen and equals 0 if mothballing is chosen. The first part of the table displays the estimated coefficients of the models while the bottom section of the table displays the average marginal effects of each of each of a set of variables related to the price change distribution and the lagged state of the well. The estimated

coefficient on *Price* is positive and significantly different from zero (0.266 and significant at 1%) as is lagged production status, *l.prod* (0.412 significant at 1%). Note that the value of “immediate investment” and “delay option” increase with price. However, increases in output price have a more substantial impact on “immediate investment” than on the “delay option.” The conclusion is intuitive. The “delayed” cash flows are discounted, and price increases should have a smaller impact on the more discounted cash flows than on the less discounted ones. So, a mothballed oil well is more likely to be reopened by oil producers when oil price increases. For a producing well, the value of the put option of “mothballing” the oil well decreases with increases in output price, making it less attractive to shut in the wells, thus leading to the positive coefficient of price. NPV analysis leads to a similar conclusion that “shutting in” the wells will reduce the value of the wells, so oil producers will not choose to shut in the oil wells. Thus, both methods predict that a producing well is less likely to be shut in if oil price increases, so the *price* coefficient should be positive.

When the lagged production status is “producing,” high expected price volatility is associated with a higher probability of choosing “producing” as the production status. That is, oil producers decide not to temporarily shut in an oil well immediately since the shut-in put option value increases, thus, it is optimal to wait (to not exercise the option) rather than to shut in immediately. Increases in price volatility increase the put option value. The fact that lagged production status as well as volatility have an impact on the choice is consistent with producers recognizing the presence of sunk costs and future uncertainty.

Price is not the only relevant variable, characteristics related to expected future production as well as costs and technologies that impact costs should play a role. I account for those characteristics. An older oil well is less likely to be in production or to produce as much in the future relative to a newer oil well. An older oil well’s oil production rate exponentially declines after it spuds.<sup>28</sup> It is also more expensive to maintain an older oil well than a newer one – both its operating cost and maintenance cost increase with age, so it is less profitable to operate a more aged oil well. The coefficient estimate on the variable *age* is negative and statistically significant, as is the marginal effect. Therefore, the probability selecting the production state is lower for older wells. The conclusion is unanimous for whether the lagged production status is “producing” or “mothballing.”

Selection of the status producing is positively related to cumulative well production (*cumulative oil production*). More productive wells are more likely to be selected to remain in production than less productive wells, as the expected production rates are higher.

Well productivity will generally depend upon the basin where the well is located. Many of the *basin* location dummies included in the model have significant coefficients ( $p < 0.10$ ), for instance, the

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<sup>28</sup> Other than if the producing oil well is re-worked which itself is costly.



productive Texas *Permian* basin. This finding suggests that it is crucial to control *basin* which potentially captures the net effect of geographic differences, including rock porosity and the age of the future potential of the location..

### 6.2.1 Skewness and Kurtosis and the Oil Production Choice

As shown earlier, the “skewness- and kurtosis-adjusted option pricing formula” predicts that increases in skewness lead to a reduction in the value of the delay option. As the delay option value declines, the immediate investment value increases, thus, oil producers are more likely to choose immediate investment. In other words, the likelihood of immediate shut down increases. Oil producers may decide to shut in wells immediately or postpone the shut-in plan. The former is determined by the “immediate investment” value (the NPV of shutting in oil wells), and the “delay option” value decides the latter. Larger skewness values lead to declines in the delay option’s value increasing the probability of immediate shut down. The impact of skewness on investment decisions is thus opposite of the effect of volatility – while an increase in volatility increases option value, a similar rise in skewness reduces option value. While higher volatility postpones investment and keeps oil wells in current status, higher skewness works in the opposite direction.

When price skewness is high, oil wells are more likely to change status than in a month when skewness is low. Thus, if value maximization prevails I predict a negative coefficient for the interaction of “producing” lagged status and skewness, and I expect a positive coefficient for that of “mothballing” lagged production status and skewness.

In column (2), the interaction of “producing” lagged status and skewness has a negative coefficient (-0.007). That coefficient on “mothballing” lagged status and skewness is positive (0.093) and significant (with a p-value < 0.001). Both results are consistent with the predictions under value maximization in the presence of the real options to produce or mothball. Higher skewness leads to acceleration of immediate investment and a stronger propensity to reopen (a mothballed oil well) or shut-in (a producing oil well). Although skewness is not statistically significant when the lagged status is “producing,” the coefficient sign is consistent with the prediction. This finding indicates that the skewness- and kurtosis-adjusted option pricing formula is a helpful tool to predict the impact of skewness (or crash risks and jumps in prices) in price distribution. These results indicate that operators make the choice to produce or mothball consistent with them considering the value of the real options involved.

Kurtosis is a measure of the price distribution's leptokurtic nature and increases with the possibility of outliers. Higher kurtosis is indicative of a distribution with fatter tails. When an option is deep-in-the-money, a higher kurtosis may be related to a higher probability of higher payoff as the result of an outlier price change. The modified option pricing formula predicts that when the option is deep-in-

the-money, an increase in kurtosis positively impacts the option's value. When an option is not deep-in-the-money, however, an increase in kurtosis (an increase in the probability of outliers) increases the likelihood of price "jumps" out of the "in-the-money" domain, which makes the option lose value. So, when the option is not deep-in-the-money, increased kurtosis harms option value and accelerates immediate investment. Thus, I predict that kurtosis has the same coefficients as skewness - the interaction between "producing" status and kurtosis should have a negative sign, and the interaction between "mothballing" status and kurtosis should have a positive sign if value maximizing behavior in the presence of these real options. When kurtosis increases, oil well status is more likely to be changed as the delay option value declines, and the immediate investment value is more likely to exceed the delay option value.

As predicted, the interaction of "producing" lagged status and kurtosis has a negative and significant coefficient (-0.192 at 1%), and the coefficient of the interaction of "mothballing" lagged status and kurtosis is positive and significant (0.173 at 1%). Increases in kurtosis accelerate investment, suggesting that an increase in kurtosis reduces the delay option value and makes it less optimal to postpone investment. The results indicate that asymmetry and dispersion have significant impacts on investment propensities. I conclude the results support the predictions that within the context of producing versus mothballing, the operators in the sample behave as if they account for skewness and kurtosis when making these decisions in a dynamic setting. These results are in addition to the influence of volatility in general.

The marginal effects for these variables are also positive and significantly different from zero.<sup>29</sup> The positive coefficient for *price* (and the positive marginal effect) indicate the probability of selecting to produce is positively related to expected prices. Likewise for the skewness and kurtosis interactions with lagged status, indicating that higher-order moments affect investment choices.

Consistent with the results in panel (1) of Table IV, the coefficient on *age* is negative and significant (-0.307 and p-value <0.001), indicating that older wells are less profitable, consistent with older wells being less productive and more costly to operate. The results on *age* are thus unanimous for the models that include and exclude higher-order moments of the price distribution. I find that the coefficient for *depth* is negative and significant too (-0.427 and p-value <0.001). The estimated coefficient on the variable cumulative oil production *cumulative oil production* is positive and statistically significant. Interestingly however the estimated coefficient on the reserve variable is not significantly different from zero. However, in unreported results excluding basin, drilling types, and depth in the empirical model, reserve becomes significant and it is positive.

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<sup>29</sup> Marginal effects are a function of the estimated model parameters (see Greene (2008, ch. 23)).

The estimated coefficient on lagged production status,  $l.prod$ , has a positive and significant coefficient. This finding suggests that lagged production status can be an essential determinant of production status. When the lagged production status is “producing,” the positive coefficient indicates that the oil well is more likely to stay in the “producing” status. Similarly, when the lagged production status is “mothballing,” the coefficient indicates that the oil well is more likely to remain in the “mothballing” status. The switching cost for changing the oil well’s production status or the sunk cost that the producers have paid to establish its current status drives this result. The higher the cost to switch from “producing” to “mothballing” (or vice versa), the less likely producers choose to change. Also, the higher the sunk cost of establishing the current production status, the less likely they will be willing to switch.

### 6.3 The Choice to Drill or Defer

Drilling an oil well costs substantially more than reopening a mothballed well. Permanently closing a well also incurs costs, but it eliminates operating and maintaining costs. I begin with an examination of the choice to drill and in the following section of the paper present an examination of the shut down decision. The examination of the drilling decision is in a spirit similar to Kellogg (2014) and Décaire et al. (2020) except that here I account for the influence of skewness and kurtosis of the price change distribution in addition to volatility. I examine the relationship between drilling an “infill oil well” and the magnitude of price distribution moments to explore the effects of price uncertainty on irreversible investment decisions, as the uncertainty affects investment options value.<sup>30</sup>

I examine the impact of the expected price and expected skewness and kurtosis on the decision to drill. Following Kellogg (2014) and Décaire et al. (2020), I examine infill wells only and use the Cox proportional hazard model. Examining infill wells allows me to focus on a set of decisions that are as reflective of new decisions as possible. To identify infill wells, I exclude the first wells in a field (the exploratory wells) and keep only the first drilled well whose spud dates are between Mar 2010 and Sep 2019 when oil price data are available in my dataset. I drop wells with spud dates before or in Mar 2010 or which are shut down after Sep 2019. Undrilled fields are identified as “unexercised options” with  $drilling=0$  through date range for all months up to the spud date at which I assign  $drilling = 1$ . There are 32,817 options among which 12,729 are exercised.

The analysis of the option to drill when considering infill drilling has the feature that as new wells are drilled the availability of new drilling options falls. As Kellogg (2014) points out a probit

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<sup>30</sup> Schlumberger describes infill drilling as “The addition of wells in a field that decreases average well spacing. This practice both accelerates expected recovery and increases estimated ultimate recovery in heterogeneous reservoirs by improving the continuity between injectors and producers. As well spacing is decreased, the shifting well patterns alter the formation-fluid flow paths and increase sweep to areas where greater hydrocarbon saturations exist.” [https://glossary.slb.com/en/terms/i/infill\\_drilling](https://glossary.slb.com/en/terms/i/infill_drilling)

specification (or for that matter the linear probability model) is therefore not the appropriate structure to account for this dynamic and Decaire et al. (2020) agree in their analyses. However, the Cox proportional hazard model does fit the setting described.<sup>31</sup>

I construct the dataset of the unexercised “drilling” options by assigning *drilling*=0 for the months between the beginning of a lease’s production and the last month before drilling. The dependent variable *drilling*=1 for the spud month. So, the months with *drilling*=0 are when the wells remain undrilled, and the months with *drilling*=1 are when drilling wells. Each well enters data when it becomes available for drilling (when the first well in the lease spuds so oil producers can prepare an infill oil well to boost oil production). This oil well has *drilling*=0 for the months after it enters the sample up to the month of drilling, but after it no longer is an opportunity as it has been drilled. Hence at any date up to drilling there is a probability that it can be drilled conditional on the economic factors prevailing at the time. Hence, this dynamic fits the Cox model nicely.

I truncate the months after the spud dates.<sup>32</sup> I report the estimation results in column (4) of Table V. The results indicate that price is positively related to the probability of drilling and that volatility is negatively associated with the likelihood. The conclusions are consistent with existing literature on real options investment. The result that price (volatility) is positively (negatively) related to the probability of drilling is consistent with the results presented in Moel and Tufano (2002), Kellogg (2014), and Décaire et al. (2020). Anderson, Kellogg and Salant (2018) find that oil well production does not respond to price changes.

I find evidence that higher-order moments influence the choice to drill. Column (4) of Table V shows that when kurtosis increases by 1.00, the probability of drilling a new oil well increases by 44.34% (exponential of 0.367). It suggests that the tail risk in the price distribution of output influences the decisions to invest. Although skewness is not significant, its coefficient (0.039) suggests that it is consistent with the prediction of the Real Options Valuation and modified option pricing models.

I also find that a well’s oil reserve is an essential determinant for drilling decisions – *reserve* has a positive and significant coefficient (0.142 and p-value <0.001). The higher the potential of being a productive oil well increases the probability of being drilled. Oil producers expect a higher production rate for wells with high reserves, and the high reserve oil wells are more likely to bring higher expected

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<sup>31</sup> The Cox proportional-hazards model (Cox, 1972) is a model used for investigating the association between the survival time of an entity, person, etc., and one or more predictor variables. Basically, it is formulated to model how a set of specified factors influence the rate of a particular event happening (e.g., drilling a well) at a particular point in time. This rate is commonly referred as the hazard rate. See Cameron and Trivedi (2005, Ch. 17) for a review of the Cox Proportional Hazard Model.

<sup>32</sup> The hazard model will not use the months after the event happens (when the dependent variable is 1). Also, the hazard model will not use any unexercised options for estimation. These observations (after exercised and never exercised options) contain no useful information for estimating the hazard model.

profits. The expected profit affects both the delay option value and the immediate investment value. However, the “immediate investment” value increases more than the “delay option,” causing a higher likelihood of drilling a high reserve oil well. The deflated drilling cost has a positive sign, but it is not statistically significant. In an unreported result, I exclude *basin*, *depth*, and *drill*, and deflated drilling cost *cost\_def* has a negative and significant coefficient. This finding suggests that the controls correlate with *cost\_def*. I expect the drilling cost to have a negative coefficient as the cost decreases project value. An increase in drilling cost is analogous to the rise in option strike price, thus reducing the delay option value. However, the immediate investment value will decrease more than the delay option value.

Next, I exclude skewness and kurtosis from the econometric model to verify the existing literature showing that volatility is negatively related to the possibility of “immediate investment.” I present the results in column (3). *Volatility* has a negative and significant coefficient (-5.453 and p-value<0.01). When controlling for *price*, *depth*, *drill*, *basin*, *reserve*, and *cost\_def*, increases in price uncertainty lead to declines in the propensity to invest immediately. Higher price uncertainty is positively related to a delay option value increase, so postponing investment is more likely to be the optimal choice for investors. I show that *price* remains a positive and significant coefficient and whose effect rarely changes after removing *skewness* and *kurtosis* from the model (1.267 and p-value<0.01). Price remains the direct impactor for investment choices – a higher price is directly related to a higher expected payoff from investing and delay option value; however, the increase in value of the “immediate investment” exceeds that in value of the option.

#### 6.4 The Choice to Permanently Shut-down (plug) or Continue Producing

I define “shutdown” (*shutdown=1*) as the last month of production for the oil well. *Shutdown=0* up until the final month of production, beginning with the in first production date and ending the previous month before closing. *Shutdown=0* when the well begins production. The well has *shutdown=1* in its last production month. I exclude the months when wells stop producing in the middle of their lives but then resume as these constitute temporary shut-ins (mothballing). The cost of reopening a permanently shut-down well is prohibitive and likewise as mentioned earlier switching to mothball status would cause a shift from no periodic maintenance cost to a per period cost, which would make no economic sense.

The choice is therefore to shut-down or to defer shutting down. The delay option value depends on the expected oil price, price volatility, skewness, and kurtosis. As the modified option pricing formula presented in Section 4 shows, when skewness increases and the call option is out-of-the-money, a call option’s value increases. By put-call parity, when the put option is in the money and skewness increases, the call option value increases, so the put option value decreases. A similar analysis can apply to the impact of kurtosis increase on put option value. The prediction is that when kurtosis increases, a deep-in-

the-money put option increases in value. Thus, I expect the put option value to decrease when skewness increases, and I anticipate that the put option value increases when kurtosis increases. If the delay option value increases and the immediate investment value remains unchanged, producers are more likely to postpone investment, as the delay option's value exceeds the value of the decision to immediately shut-in the well. Suppose the delay option's value decreases and the "immediate investment" value remains unchanged. In that case, maximizing producers make the immediate decision to close the well. So, when skewness increases, producers are more likely to select to close. When kurtosis increases and the option is deep-in-the-money, producers are more likely to postpone the shutdown choice. Thus, I posit that the probability of the choice to shut down is increasing in skewness. Likewise, the probability of closing is negatively related to kurtosis if the shutdown option is deep-in-the-money. I have made such predictions in the hypothesis development section.

The empirical results are unsurprisingly consistent with these predictions. I report these results in column (6) of Table VI. The results indicate that price is negatively related to the probability of closing an oil well. I showed that price is positively related to the likelihood of drilling, increasing the expected payoffs from drilling an oil well. Such value increases are more for "immediate investment" than "delay option," as the latter is discounted more by time. Similarly, when price decreases, oil producers are more likely to choose to close down the oil wells, concerning that the declines in oil price will make an operating oil well unprofitable. So, when the opposite happens, oil producers are less likely to close down the oil wells. The oil producers have two choices – the first is to shut down the oil wells immediately, and the second is to postpone the shutdown plan. Both become less attractive as oil price increases; however, the first choice is even less appealing to producers as the increases in oil price affect the expected payoffs from the first choice more than the second one. So, when oil price increases, oil producers are more likely to postpone shutdown plans. I also find that the coefficient of *volatility* is negative, indicating increased expected volatility leads to an increased value for the delay option and has a lower probability of selecting to shut-in. When price volatility is high, there is a better chance that, in the future, a price increase will make the shutdown plan unattractive. So, if increases in price volatility increase the delay option value, producers tend to choose to delay shutting down wells. I should obtain a negative coefficient for volatility, and the empirical results suggest that *volatility* is negatively related to the probability of shutdown, verifying ROV predictions. To summarize, the empirical results by far with price and volatility are consistent with existing literature and verify the validity of Real Options Valuation in solving irreversible investment problems.

More importantly, I find that skewness and kurtosis are positively and negatively related to the probability of oil well shutdowns, respectively, which conclusion is consistent with the prediction of Real Options Valuation. In column (6), the coefficient of skewness is significant at the 1% level. It suggests

that, when increases in price skewness, i.e., increases in the probability of positive jumps in prices, producers are more likely to shut down oil wells immediately. This finding seems counter-intuitive at first; however, the put option value decreases, suggesting that it is not worth waiting to proceed with the shutting down plan. The expected payoff from the immediate shutdown does not change with skewness but is related to the expected prices. The result is consistent with the immediate shutdown value being perceived as greater than the value of waiting. The coefficient of kurtosis is not statistically significant; however, its sign (-0.083) is consistent with the prediction that kurtosis increases the delay put option value. Increases in the probability of price outliers reduces the value of the immediate decision. To summarize, I find that extreme positive shocks accelerate a close-down plan, and outliers on both sides decrease the value from making such decisions immediately.

I also find that (oil well) depth is positively related to the probability of shutdown. Deeper wells have higher operating costs than shallower wells; thus, it is more likely to become optimal to shut down. I include *basin* in the econometric model, showing that the dummies are significant explanatory variables for investment decisions. I use *last12* as a proxy for the productivity of the well. If the oil well remains productive, its expected payoffs from future production are higher than a less effective oil well. I show that *last12*, i.e., the well's last 12-month's production rate, is negatively associated with the propensity to close down the well. An oil well's production rate decays with time, and its decline is exponential. Thus, even for the same well, its profitability changes with time. A more productive and newer well is more profitable to keep in the production state than a less effective and older well. Similar to what I did before, I ran a similar regression excluding skewness and kurtosis to explore the validity of ROV in explaining irreversible investments in the existing literature in column (5). I show that both price and volatility are negatively associated with the probability of shutting down wells (as reported in column (5)), which is consistent with the full model's results. The coefficients of other variables largely remain unchanged.

## 6.5 Preference Theory

If managers (decision makers) slant the investment decisions I examine to suit their own personal preferences then the relationship between skewness, kurtosis, and real options investments could be driven by personal investment preferences for skewness and kurtosis. If that was the case the empirical results should have been different from what I find. If managers have a personal preference for positive skewness and that colors the decisions I study, then I should have observed that oil wells are more likely to be drilled or operations resumed when skewness is more positive and that they are more likely to be closed or mothballed when skewness is more negative. If, however, investors have a preference over negative skewness, oil wells are more likely to be drilled or resumed when skewness is more negative and they are less likely to be closed or mothballed when skewness becomes more negative. My empirical

results show the relations between skewness, kurtosis, and real investments of oil wells is consistent with the predictions of the Real Options Valuation and the modified Black-Scholes option pricing model. This finding suggests that choices studied are consistent largely with value maximizing behavior as laid out in Section 4.

## **6.6 Summarizing the Results**

The empirical results presented in Tables VI, V and VI are found to generally be consistent with the hypothesis that production versus temporary shut down decisions, drilling versus waiting to drill and permanent shutdown versus continuing to produced choices are consistent with value maximizing behavior in the presence of real options (price uncertainty and sunk costs). When comparing the full model (that includes skewness and kurtosis) and the volatility model (everything the same but excluding skewness and kurtosis), I present evidence that the model that includes skewness and kurtosis, in addition to volatility, better explains the changes between the “producing” and the “mothballing” status of oil wells. The choice to drill or delay drilling, while determined in part by expected price change volatility, is influenced only by kurtosis not skewness. Likewise, the choice to continue production or permanently shutting down a well is determined by skewness and volatility but kurtosis does not play a role. The results are found to be robust to alternative specifications of the models to which I now turn.

## **6.7 Robustness of the Results**

### **6.7.1 Unobserved Heterogeneity in the Production/Temporary Shut-in (Mothball) Model**

I investigate whether unobserved heterogeneity across wells affects the main results presented in Tables IV-VI. There may exist substantial unobserved heterogeneous differences among oil wells that determine production statuses. Determinants other than depth, age, drilling type, or basin may affect the profitability of the oil wells, such as operating costs. Using a robust model controlling for unobserved heterogeneity, I implement the simple initial condition solution proposed by Wooldridge (2005) to solve the unobserved heterogeneity issue in random-effects panel probit model. In addition to the initial model’s specification, I include add the lagged values (initial conditions) of the dependent variable, cumulative oil production, and age’s initial values, and averages of cumulative oil production and age to control unobserved heterogeneity among wells. I find that the empirical results rarely change. I obtain positive and significant coefficients for lagged production status, price, the interaction of volatility and “producing” lagged status, the interaction term of skewness and “mothballing” lagged production status, the interaction term of kurtosis and “mothballing” lagged status, and cumulative oil production. Also, I continue to obtain negative and significant coefficients for the interaction term of kurtosis and “producing” lagged status, age, and depth. This result suggests that my original model in section 6.2 does not suffer substantially from the unobserved heterogeneity issue.



### **6.7.2 The Influence of Nearby Drilling Activity**

Décaire et al. (2020) find that the propensity for oil producers to decide whether to drill a new oil well is influenced by the drilling activity of neighbor producers, suggesting that an information spillover effect influences producers decisions to drill. I construct measures of proximity in the following manner: using the arrangement of sections within a lease,<sup>33</sup> I can identify the “neighbors” of an oil well with the section numbers. A section can have up to eight neighbors. For instance, section 16 has neighbor sections 8, 9, 10, 15, 17, 20, 21, and 22. However, some sections have fewer neighbors within the lease. For instance, section 1 only has neighbor sections 2, 11, and 12. A well lease can be identified by its range and township values which, along with section numbers, are available from the Enverus’ database. After identifying each well’s neighbors, I count the number of drilled options among these neighbors. These are the number of exercised options around the well. This number serves as a source of spillover information, as it indicates the productivity of the oil wells and reserve underground. I am able to split between the oil well’s firm’s own drilled neighboring wells and drilled wells from other firms. Both numbers are included in the empirical model to examine the effect of proximity options exercises on oil well drilling decisions.

Including these measures of proximity in the base drilling model, I find that my basic results regarding the relation between the choice to drill, volatility, skewness and kurtosis are unchanged. However, I do find that proximity does matter in my sample, that the probability of drilling is positively related to proximity.

### **6.7.3 The Regulatory Environment of the State in Which a Well is Located**

While the activities I have been studying are all potentially affected by U.S. Federal oversight and regulation, the regulatory environments may differ across the states included in the sample (Texas, Oklahoma, California, North Dakota and Pennsylvania). The models evaluated to this point control for the basin location of wells, that only indirectly controls for the state regulatory environment. Evaluation of these states indicates that the regulatory environment is significantly more restrictive in California relative to the other states represented in the sample. I divided the sample into two subsamples, one which includes only activity in California and the second which includes all the remaining states. The variable *reserve* is replaced by the first 12-months’ production rate (*first12*) as reserve is not available for California oil wells. I repeat the estimation of the drilling model for each subsample. The results with only California oil wells and excluding California oil wells do not differ.

### **6.7.4 Did the Focus on Environmental, Social And Governance Issues (ESG) Impact the Production, Drilling and Shut-down Decisions?**

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<sup>33</sup> An example of the map of the arrangements of section within a well lease can be accessed: <https://www.geomore.com/locating-wells/>.

I end by separating the sample into two time periods. Up to the end of 2015 and after. The choice of point at which to divide the sample period is based upon the Paris Climate Agreement that was adopted by 196 Parties at COP 21 in Paris, on 12 December 2015 and entered into force on 4 November 2016.<sup>34</sup> Inspection of the Google Trend Index based upon the phrase ‘ESG’ shows there was a sharp escalation in attention to ESG issues following 2015 and so seems to be a natural point of demarcation. Oil and natural gas producers were directly or indirectly impacted by the adoption of this accord and may as a consequence have significantly altered how they make decisions. I find that the ESG shift has no impacts on the effects of higher-order moments (skewness and kurtosis) of price distribution of oil after 2015 – the coefficients on skewness and kurtosis are still consistent with the Real Options Valuation and modified option pricing models’ predictions.

## **7. Summary and Conclusions**

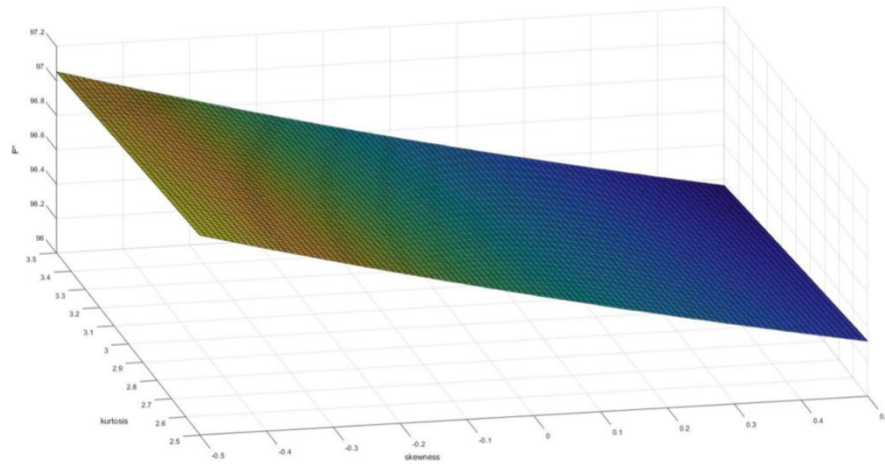
In this study, I show that implied skewness and implied kurtosis significantly affect irreversible investment decisions by finding the association between oil price implied skewness and kurtosis and oil well production. By focusing on production histories for all oil wells domiciled in California, North Dakota, Pennsylvania, Texas, and Oklahoma from 2010 to 2019 and relating the changes in producing status to oil price moments (volatility, skewness, and kurtosis), I show that oil producers account for higher order oil price moments consistent with the predictions of value maximizing behavior in the presence of real options (price change uncertainty and sunk costs). The choices to produce or temporarily shut down, are determined by expected skewness, kurtosis and volatility of the price change distribution in addition to other well-specific characteristics. The choice to continue producing or permanently shut down are determined by expected skewness and volatility. The choice to drill or wait to drill is determined however only by expected kurtosis and volatility. The evidence overall indicates that sunk costs matter in the presence of future price uncertainty. The choices of the oil producers in the sample are consistent with value maximizing behavior in the presence of real options, however not all choices are influenced by skewness and kurtosis despite the fact that in theory the value of the real options involved are influenced by those parameters of the price change distribution.

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<sup>34</sup> See <https://unfccc.int/process-and-meetings/the-paris-agreement/the-paris-agreement>

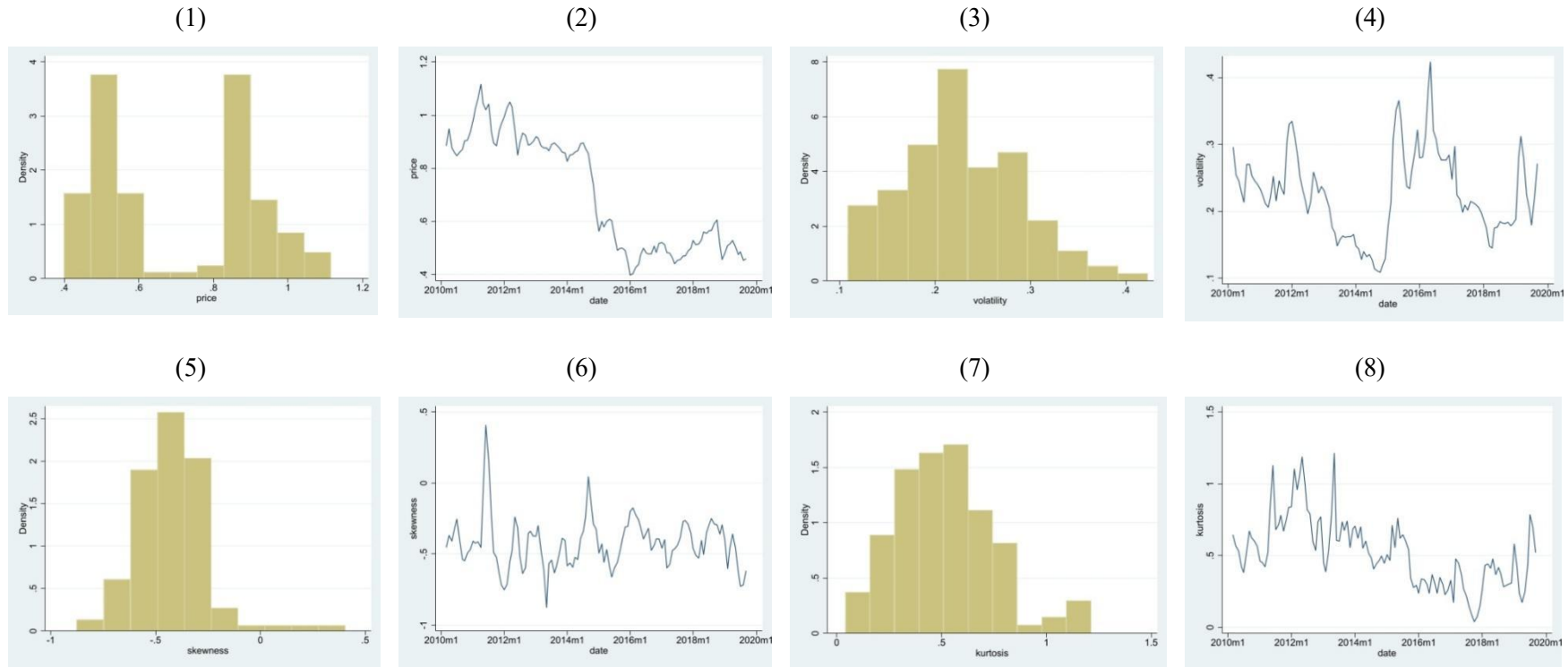
### FIGURE I. Optimal Exercise Prices of A Call Option Upon Changes in Skewness and Kurtosis in Underlying Security Price Return Distribution

Figure I. shows the optimal exercise prices of a call option for different values of skewness and kurtosis in the underlying security price return distribution. The call option value is calculated using the modified Black-Scholes option pricing model. Immediate exercise value is calculated using the conventional Black-Scholes option pricing model. On the x-axis, skewness changes from -0.5 to +0.5 in 0.01 tick, and on the y-axis, kurtosis increases from 2.5 to 3.5 for the changes in excess kurtosis from -0.5 to +0.5 in 0.01 tick. Each fixed skewness-kurtosis values is simulated for 10,000 times for underlying price changes from 92.0001 to 112.00. The exercise threshold is when the difference between call option value and immediate exercise value is lower than 0.01. Z-axis shows the optimal exercise price of the call option. The optimal exercise price of the call option is simulated using Stata and graphed in Matlab. The figure shows that the optimal exercise of the call option declines as skewness in underlying security prices increases. The optimal exercise price does not have an apparent change as kurtosis in underlying price increases. I assume a constant annual risk-free rate 2%, annual standard deviation in underlying security prices 20%, time interval one month, dividend yield 6%, and strike price 92.00.



**FIGURE II. Oil Price and Central Moments**

Figure II. (1) and (2) show the histogram of three-month lagged 18-month oil futures price and graph of the price to date, respectively; (3) and (4) shows the histogram of 18-month three-month lagged option-implied volatility and graph of the volatility to date, respectively; (5), (6), (7), and (8) shows the histograms of 18-month three-month lagged option-implied skewness and kurtosis and the graphs of the central moments to dates, respectively. Oil price is scaled by 100 and deflated by CPI.



**TABLE I. Coefficient Sign Predictions of Choice Determinants for Choice Models**

Table I. shows the predicted relationships between the empirical model variables as discussed in sections 4.3.1, 4.4.1, and 4.5.1. and the probability of oil producers choosing “producing” (“drilling” or “shutdown”) status at  $t$  according to the developed economic and empirical models in 4.3, 4.4, and 4.5. These relationships are indicated by the modified Black-Scholes option pricing model and Real Options Valuation as discussed in section 4.1. The first column shows the names of the variables in these empirical models (as in equations (1), (2), and (3)). Columns 3-5 show the predicted relationships between the variables and the probability of choosing “producing” as in the production model, and the probability to choose “drilling” in the drilling model, as well as the probability to choose to close the oil well in the shutdown model. In particular, the relationship between the probability and price moments for the production model depends on lagged status, so column 2 gives the lagged status of oil wells. In the second column, lagged producing status is given as signs of predicted coefficients depending on lagged status.

Model		Production	Drilling	Shutdown
Variable	Lagged Production State at t-1	Probability of Selecting “Producing” State at t	Probability of Selecting “Drilling” at t	Probability of Selecting “Shutdown” at t
<i>Lagged Status(= “Producing”)</i>		Positive		
<i>Price</i>		Positive	Positive	Negative
<i>Volatility</i>	<i>Producing</i>	Positive	Negative	Negative
	<i>Mothballing</i>	Negative		
<i>Skewness</i>	<i>Producing</i>	Positive	Positive	Positive
	<i>Mothballing</i>	Negative		
<i>Kurtosis</i>	<i>Producing</i>	Positive or Negative	Positive or Negative	Positive or Negative
	<i>Mothballing</i>			
<i>Horizontal Drilling</i>		Positive or Negative	Positive or Negative	Positive or Negative
<i>Directional Drilling</i>		Positive or Negative	Positive or Negative	Positive or Negative
<i>Undetermined Drilling</i>		Positive or Negative	Positive or Negative	Positive or Negative
<i>Basins</i>		Positive or Negative	Positive or Negative	Positive or Negative
<i>Age</i>		Negative		
<i>Depth</i>		Negative	Negative	Positive
<i>Productivity</i>		Positive	Positive	Negative
<i>Total Costs</i>		Positive	Negative	Negative

**TABLE II. Life Cycle of An Oil Well**

Table II. describes the six stages of the life cycle of an oil well – 1. Seismic survey; 2. Drilling; 3. Producing; 4. Mothballing, 5, Producing (Resuming), and 6. Shutdown. The second column describes the definition of each stage and the last column explains the situations in which each stage would be chosen.

Oil well life cycle stage	Define	When to choose the stage
1. Seismic Survey	Tests of oil well expected total production and operating cost	Oil producers to test oil reserve and reservoir depth
2. Drilling	Use rigs to open a new well and stabilize wellbore with cement and steel	Reservoir is detected; maintaining leasing a field
3. Producing	Pump up crude oil products and sell at market (future) prices	Expected revenue from selling is higher than operating cost; well is not exhausted or dry
4. Mothballing	Temporarily stop production of an oil well but maintain the option to re-open it later	Expected revenue is lower than operating cost but may becomes greater than cost in the future
5. Producing (Resuming)	Resume production after mothballing when prices are better	Resume production of oil well if expected revenue becomes greater than costs
6. Shutdown	Permanently close a well by plugging in wellhead with cement	Well is exhausted; too costly to maintain an active oil well

**TABLE III. Statistical Summary**

Data consists of oil wells in California, Pennsylvania, North Dakota, Oklahoma, and Texas. Dates range monthly from Jan 2010 to Jun 2019. “Prod”=1 for producing well-month observations and “Prod”=0 for mothballing. Age is the number of months in production from the completion date. Cum. oil production indicates the cumulative oil production measured in bbl. Price is the one-month maturity WTI futures prices (scaled by 100 and deflated by CPI). Volatility, skewness, and kurtosis are BKM’s (Bakshi et al. (2003)) option-implied risk-neutral central moments computed from WTI crude oil futures prices and option prices. *Reserve* is the total tested oil potential in bbl. *Depth* is in tens of thousands of feet. Drilling type dummies include directional, horizontal, vertical, and undetermined drilling types. *Last12* is the oil well’s last 12-months’ production rate and it is only for the shutdown dataset. *Exer(drilling)* (*exer(shutdown)*) is the drilling variable in the drilling(shutdown) dataset and it is equal to one for spud (shutdown) month. The variables *age*, *cum. oil production*, *reserve*, *depth*, and *last12* are taken natural logarithm. The last panel summarizes the dummies for the wellheads’ located basins.

Variable	Obs	Mean	Std. dev.	Min	Max	Basins	Obs	Percent	Cumulative
<i>prod</i>	53,135,284	0.6239	0.4844	0.0000	1.0000	<i>Anadarko</i>	3,507,419	7.04	7.04
<i>age</i>	53,135,284	4.8450	1.0263	0.0000	6.6644	<i>Appalachian</i>	9,273,059	18.60	25.64
<i>cum. oil production</i>	42,251,592	8.8826	2.8138	-73.4215	16.4272	<i>Ardmore</i>	135,499	0.27	25.91
<i>price</i>	53,135,284	0.7147	0.2097	0.3977	1.1064	<i>Ark-la-tx</i>	2,772,800	5.56	31.47
<i>kurtosis</i>	53,135,284	0.1636	0.3108	-0.4402	1.0916	<i>Arkoma</i>	1,002,489	2.01	33.48
<i>skewness</i>	53,135,284	-0.2318	0.1882	-0.6808	0.6169	<i>Cherokee platform</i>	832,007	1.67	35.15
<i>volatility</i>	53,135,284	0.2145	0.0603	0.1040	0.4042	<i>Delaware</i>	672,646	1.35	36.50
<i>reserve</i>	6,932,426	4.2573	1.6864	-4.6052	9.7365	<i>Fort worth</i>	5,427,499	10.89	47.39
<i>depth</i>	21,521,972	8.2495	0.8085	2.9957	10.1266	<i>Gom offshore</i>	70,442	0.14	47.53
<i>last12</i>	25,722,321	6.0047	2.1597	0.0000	13.9951	<i>Los angeles</i>	444,786	0.89	48.42
<i>exer(drilling)</i>	5,084,240	0.3281	0.4695	0.0000	1.0000	<i>Marietta</i>	66,390	0.13	48.56
<i>exer(shutdown)</i>	27,391,704	0.0089	0.0941	0.0000	1.0000	<i>Mid-continent other</i>	383,672	0.77	49.32
<b>Drilling Types</b>	<b>Obs</b>	<b>Percent</b>	<b>Cumulative</b>			<i>Midland</i>	4,383,562	8.79	58.12
<i>directional</i>	2,127,199	4.00	4.00			<i>Palo duro</i>	36,534	0.07	58.19
<i>horizontal</i>	7,166,473	13.49	17.49			<i>Permian other</i>	7,546,061	15.14	73.33
<i>undetermined</i>	170,977	0.32	17.81			<i>Sacramento</i>	116,535	0.23	73.56
<i>vertical</i>	43,670,635	82.19	100.00			<i>Santa maria</i>	115,984	0.23	73.80
<b>State</b>	<b>Obs</b>	<b>Percent</b>	<b>Cumulative</b>			<i>Ventura</i>	235,107	0.47	74.27
<i>California</i>	5,790,727	10.90	10.90			<i>Western gulf</i>	6,812,855	13.67	87.93
<i>North Dakota</i>	1,245,491	2.34	13.24			<i>Western us other</i>	4,769,578	9.57	97.50
<i>Oklahoma</i>	5,833,824	10.98	24.22			<i>Williston</i>	1,245,491	2.50	100.00
<i>Pennsylvania</i>	9,273,059	17.45	41.67						
<i>Texas</i>	30,992,183	58.33	100.0						

**TABLE IV. The Choice to Produce or Mothball**

TABLE IV. Estimation results of producing status on price, central moments, and controls using the random-effects panel probit model. The regressions examine the impacts of the determinants on exercise of real options of oil well production. The model specification in Equation (1). The dependent variable, *prod*, =1 for “producing” and =0 for “mothballing”. *L.prod* =1: lagged producing status is “producing” and =0: lagged producing status is “mothballing”. The option-implied risk-neutral central moments (volatility, skewness, and kurtosis) are computed using BKM (2003). Well depth is measured in tens of thousands of feet, and well age is measured in months. “horizontal drilling” and “vertical drilling” are drilling types and “directional drilling” is the benchmark. “Ardmore”, “Delaware”, ... are basin dummies. The variables, *age*, *depth*, *reserve*, and *cumulative oil production* are taken natural logarithms. The reported  $\rho$  is the proportion of panel-level variance in the total variance of dependent variable. \*, \*\*, and \*\*\* indicate significance at 10%, 5%, and 1%, respectively.

	(1)			(2)		
*prod=1 for producing and prod=0 for mothballing						
dependent variable: prod	coefficient	robust SE	p	coefficient	robust SE	p
<i>l.prod</i>	0.412	0.028	0.000 ***	0.376	0.029	0.000 ***
<i>price</i>	0.266	0.031	0.000 ***	0.229	0.033	0.000 ***
<i>{l.prod=0} × vol</i>	0.030	0.091	0.739	-0.115	0.083	0.166
<i>{l.prod=1} × vol</i>	0.266	0.056	0.000 ***	0.428	0.061	0.000 ***
<i>{l.prod=0} × skewness</i>				0.093	0.015	0.000 ***
<i>{l.prod=1} × skewness</i>				-0.007	0.013	0.580
<i>{l.prod=0} × kurtosis</i>				0.173	0.019	0.000 ***
<i>{l.prod=1} × kurtosis</i>				-0.192	0.014	0.000 ***
<i>age</i>	-0.307	0.021	0.000 ***	-0.318	0.021	0.000 ***
<i>depth</i>	-0.427	0.042	0.000 ***	-0.424	0.042	0.000 ***
<i>reserve</i>	-0.008	0.017	0.625	-0.009	0.017	0.602
<i>cumulative oil production</i>	0.198	0.029	0.000 ***	0.197	0.029	0.000 ***
<i>horizontal drilling</i>	0.828	0.067	0.000 ***	0.811	0.067	0.000 ***
<i>vertical drilling</i>	-0.280	0.063	0.000 ***	-0.738	0.197	0.000 ***
<i>undetermined</i>	-0.750	0.194	0.000 ***	-0.270	0.063	0.000 ***
<i>Ardmore</i>	-0.207	0.052	0.000 ***	-0.204	0.051	0.000 ***
<i>Ark-la-TX</i>	1.938	0.097	0.000 ***	1.929	0.097	0.000 ***
<i>Arkoma</i>	-0.725	0.084	0.000 ***	-0.725	0.085	0.000 ***

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TABLE IV. The Choice to Produce or Mothball (continued)

	(1)			(2)		
*prod=1 for producing and prod=0 for mothballing						
dependent variable: prod	coefficient	robust SE	p	coefficient	robust SE	p
<i>Cherokee platform</i>	-0.479	0.039	0.000***	-0.478	0.039	0.000***
<i>Delaware</i>	1.679	0.052	0.000***	1.663	0.051	0.000***
<i>Fort Worth</i>	2.003	0.059	0.000***	1.989	0.059	0.000***
<i>Gom offshore</i>	2.116	0.384	0.000***	2.118	0.386	0.000***
<i>Marietta</i>	0.016	0.096	0.864	0.022	0.096	0.821
<i>mid-continent other</i>	-0.316	0.036	0.000***	-0.316	0.036	0.000***
<i>Midland</i>	3.025	0.043	0.000***	3.001	0.043	0.000***
<i>Palo Duro</i>	1.533	0.191	0.000***	1.520	0.191	0.000***
<i>Permian other</i>	3.159	0.059	0.000***	3.144	0.060	0.000***
<i>western gulf</i>	1.789	0.036	0.000***	1.775	0.036	0.000***
<i>Williston</i>	0.140	0.029	0.000***	0.161	0.029	0.000***
<i>constant</i>	3.391	0.333	0.000***	3.444	0.334	0.000***
p		0.650		0.648		
<i>pseudo R-squared</i>		0.058		0.059		
<i>likelihood test ratio</i>		0.000		0.000		
<i>obs</i>		6,452,102		6,452,102		
<i># of wells</i>		99,041		99,041		
<i>log likelihood</i>		-766,151.54		-765,221.33		
<i>marginal effects</i>	<i>dy/dx</i>	<i>SE</i>	<i>p</i>			
<i>price</i>	0.015	0.002	0.000***			
<i>{l.prod=0} × vol</i>	-0.008	0.006	0.165			
<i>{l.prod=1} × vol</i>	0.037	0.005	0.000***			
<i>{l.prod=0} × skewness</i>	0.006	0.001	0.000***			
<i>{l.prod=1} × skewness</i>	-0.007	0.001	0.000***			
<i>{l.prod=0} × kurtosis</i>	0.012	0.001	0.000***			
<i>{l.prod=1} × kurtosis</i>	-0.025	0.002	0.000***			

**TABLE V. The Choice to Drill or Defer**

TABLE V. panels (3)-(4) show results of the regression of the real option exercise on price, central moments, and controls using the Cox hazard model. The regressions examine the impacts of the determinants on exercise of real options of oil well entry. The regression equation is Equation (2). The dependent variable “exer” dummy=1 for “drilling” months and =0 for the months before exercising of the options. The option-implied risk-neutral central moments (volatility, skewness, and kurtosis) are computed using BKM (2003). Well depth is measured in tens of thousands of feet. “horizontal drilling” and “directional drilling” are drilling types. “Arkoma”, “east Texas”, ... are basin dummies. Drilling costs, *cost\_def*, are sums of drilling, completion, tie-in, and transportation costs and are deflated. The variables, *depth*, *cost\_def*, and *reserve* are taken natural logarithms. \*, \*\*, and \*\*\* indicate significance at 10%, 5%, and 1%, respectively.

	(3)				(4)			
*drilling=1 for spud month and =0 for prior months								
dependent variable:		robust				robust		
drilling	coefficient	SE	p		coefficient	SE	p	
<i>price</i>	1.267	0.076	0.000	***	1.000	0.101	0.000	***
<i>volatility</i>	-5.453	0.211	0.000	***	-5.587	0.216	0.000	***
<i>skewness</i>					0.039	0.083	0.636	
<i>kurtosis</i>					0.367	0.090	0.000	***
<i>depth</i>	-0.286	0.059	0.000	***	-0.291	0.069	0.000	***
<i>cost_def</i>	0.032	0.048	0.000	***	0.036	0.048	0.453	
<i>reserve</i>	0.141	0.011	0.002	***	0.142	0.011	0.000	***
<i>horizontal drilling</i>	-0.038	0.090	0.085	*	-0.039	0.090	0.665	
<i>vertical drilling</i>	0.153	0.083	0.000	***	0.151	0.083	0.070	*
<i>Arkoma</i>	-1.037	0.192	0.000	***	-1.034	0.192	0.000	***
<i>Cherokee platform</i>	-0.456	0.078	0.000	***	-0.454	0.078	0.000	***
<i>Fort Worth</i>	0.194	0.064	0.002	***	0.195	0.064	0.002	***
<i>Hollis-Hardeman</i>	0.230	0.133	0.085	*	0.234	0.134	0.081	*
<i>Central basin platform</i>	0.835	0.058	0.000	***	0.839	0.058	0.000	***
<i>Delaware</i>	0.752	0.052	0.000	***	0.750	0.052	0.000	***
<i>East Texas</i>	-0.263	0.116	0.023	**	-0.263	0.116	0.023	**

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**TABLE V. The Choice to Drill or Defer (continued)**

(3)				(4)				
*drilling=1 for spud month and =0 for prior months								
dependent variable:		robust			robust			
drilling	coefficient	SE	p		coefficient	SE	p	
<i>Eastern shelf</i>	0.161	0.092	0.079	*	0.162	0.092	0.077	*
<i>Gulf coast central</i>	-0.020	0.077	0.794		-0.020	0.077	0.799	
<i>Gulf coast west</i>	0.558	0.040	0.000	***	0.554	0.040	0.000	***
<i>Kerr</i>	1.055	0.250	0.000	***	1.064	0.253	0.000	***
<i>Llano uplift</i>	0.149	1.067	0.889		0.159	1.072	0.882	
<i>Midland</i>	0.738	0.040	0.000	***	0.740	0.040	0.000	***
<i>Northwest shelf</i>	0.149	0.218	0.494		0.152	0.218	0.487	
<i>Palo Duro</i>	0.194	0.211	0.358		0.194	0.211	0.358	
<i>Val Verde</i>	0.859	0.378	0.023	**	0.864	0.379	0.023	**
<i>pseudo R-squared</i>		0.016			0.016			
<i>likelihood test ratio</i>		0.000			0.000			
<i>obs</i>		1,203,762			1,203,762			
<i># of wells</i>		32,817			32,817			
<i>log likelihood</i>		-120,328.90			-120,307.99			

**TABLE VI. The Choice to Shutdown or Continue Producing**

TABLE VI. panels (5)-(6) show results of the regression of the real option exercise on price, central moments, and controls using the Cox hazard model. The regressions examine the impacts of the determinants on exercise of real options of oil well exits. The regression equation is Equation (3). The dependent variable “exer” dummy=1 for “shutdown” months and =0 for the months before exercising of the options. The option-implied risk-neutral central moments (volatility, skewness, and kurtosis) are computed using BKM (2003). Well depth is measured in tens of thousands of feet and taken natural logarithm. “horizontal drilling” and “directional drilling” are drilling types. “Arkoma”, “east Texas”, ... are basin dummies. Drilling costs, *cost\_def*, are sums of drilling, completion, tie-in, and transportation costs and are deflated. The variables *cost\_def* and *last12* are taken natural logarithms. \*, \*\*, and \*\*\* indicate significance at 10%, 5%, and 1%, respectively.

	(5)				(6)			
*shutdown=1 for shutdown month and =0 for prior months								
dependent variable:	robust				robust			
shutdown	coefficient	SE	p		coefficient	SE	p	
<i>price</i>	-4.782	0.098	0.000	***	-4.498	0.147	0.000	***
<i>volatility</i>	-8.010	0.300	0.000	***	-7.389	0.316	0.000	***
<i>skewness</i>					0.696	0.139	0.000	***
<i>kurtosis</i>					-0.083	0.109	0.444	
<i>depth</i>	0.235	0.034	0.000	***	0.234	0.034	0.000	***
<i>cost_def</i>	-0.091	0.020	0.000		-0.090	0.020	0.000	***
<i>last12</i>	-0.090	0.009	0.000	***	-0.090	0.009	0.000	***
<i>horizontal drilling</i>	-1.317	0.110	0.000	***	-1.316	0.110	0.000	***
<i>vertical drilling</i>	-0.229	0.080	0.004	***	-0.228	0.080	0.004	***
<i>undetermined</i>	0.013	0.114	0.909		0.013	0.114	0.909	
<i>central basin platform</i>	-0.151	0.107	0.157		-0.151	0.107	0.157	
<i>Delaware</i>	0.217	0.115	0.060	*	0.217	0.115	0.059	*
<i>east Texas</i>	0.280	0.082	0.001	***	0.280	0.082	0.001	***
<i>east Texas costal</i>	1.148	0.244	0.000	***	1.148	0.244	0.000	***
<i>eastern shelf</i>	-0.092	0.131	0.484		-0.092	0.131	0.481	
<i>Fort Worth</i>	0.925	0.259	0.000	***	0.923	0.158	0.000	***
<i>gulf coast central</i>	0.817	0.154	0.000	***	0.816	0.154	0.000	***
<i>gulf coast west</i>	0.671	0.127	0.000	***	0.671	0.127	0.000	***
<i>Hollis-Hardeman</i>	0.437	0.169	0.010	**	0.437	0.169	0.010	**
<i>Kerr</i>	-0.267	0.240	0.313		-0.267	0.264	0.312	
<i>Llano uplift</i>	-0.378	0.457	0.408		-0.380	0.457	0.406	
<i>midland</i>	0.005	0.100	0.957		0.005	0.100	0.961	

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**TABLE VI. The Choice to Shutdown or Continue Producing (continued)**

	(5)			(6)				
*shutdown=1 for shutdown month and =0 for prior months								
dependent variable:	robust			robust				
shutdown	coefficient	SE	p		coefficient	SE	p	
<i>northwest shelf</i>	-0.419	0.119	0.000	***	-0.419	0.119	0.000	***
<i>Palo Duro</i>	0.093	0.248	0.709		0.092	0.248	0.710	
<i>Val Verde</i>	1.764	0.57	0.002	***	1.768	0.571	0.002	***
<i>pseudo R-squared</i>	0.064				0.064			
<i>likelihood test ratio</i>	0.000				0.000			
<i>obs</i>	10,513,982				10,513,982			
<i># of wells</i>	135,034				135,034			
<i>log likelihood</i>	-227,430.25				-227,318.36			

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## **Appendix A. Computation of BKM Option-Implied Risk-Neutral Central Moments**

I obtain crude oil (symbol: CL) options on futures prices and futures prices from the CME Group Datasets (End of Day Complete database). Only option prices with maturities between 10 and 180 days are used, which are the most liquid options contracts. I manually dropped some outliers when the prices is 100 times greater than average, which is identified as data input error. Risk-free rates are obtained from the OptionMetrics files. Risk-free rates are extrapolated and interpolated to ensure sufficient risk-free rates for maturities of 10-180 days. I follow Chang, Christoffersen, and Jacobs (2013) and Ruan and Zhang (2018), when computing the risk-neutral central moments. I filter data by dropping option prices lower than 3/8 (minimum tick) and deep-in-the-money options (put options with an exercise price higher than 103% of futures price and call options with an exercise price lower than 97% of futures price) as well as drop days with fewer than 2 puts or calls prices, and the options prices violating the spot-futures arbitrage condition. I expand moneyness for each date and maturity to the range between 0.0001 and 3 and use the implied volatility (from CME's modified Black-Scholes option pricing model) to interpolate and extrapolate implied volatilities in that range. Next, I use the implied volatility to infer option prices using the Black-Scholes options on futures model to obtain a smooth option price in the moneyness 0.0001 and 3 for each date and maturity. With futures price and option prices in the moneynesses range [0.0001,3], I calculate option implied volatility, skewness, and kurtosis following Bakshi, Kapadia, and Madan (2003) using the trapezoidal integration following Chang, Christoffersen, and Jacobs (2013) and Ruan and Zhang (2018). I then compute the 18-month BKM risk-neutral central moments. I use futures prices to estimate the term structure of oil return realized central moments, then use the 10-180 days central moments to interpolate the one-month maturity central moments. I then use the one-month central moments and term structure to extrapolate 18-month maturity central moments. The last step is to take three-month lags of the central moments for drilling and shutdown model moments. The methods are similar to those employed by Kellogg (2014) when computing forward looking implied volatilities.