The Benchmark Curves Calibration approach: a climate scenario-based methodology to asses the impact of climate change on interest rates

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Abstract

The aim of the paper is the assessment of the effects of climate risk on interest rates by implementing a new methodology that we called *Benchmark Curves Calibration* approach (BCC for short), which is coherent with a forward-looking evaluation based on climate scenarios. The BCC approach is applied to a climate version of Vasicek model, whose calibration is based on the setting up of benchmark yield curves affected by an exogenous variation (depending on climate scenarios and drivers' correlations), which allow to propagate the effects by a reverse bootstrapping. The empirical analysis performed shows the movements of the term structure whose dimension reflect the variation in uncertainty and risk perceived by the market. The BCC approach is actually discovered as an useful policy tool, able to facilitate the market's transition toward the aimed EU goals.

1 Introduction

Currently, it is clear that climate change is affecting our entire world. Numerous evidences show that the atmosphere, the oceans, the cryosphere and the biosphere, have all changed largely due to the human influence in the past years. It is unequivocal that anthropogenic activities led to a constant rise in well-mixed greenhouse gas (GHG) concentrations [6]. There is no denying that the atmosphere is significantly warmer and the climate is changing over the years. And this comes with consequences. Extreme weather events are more frequent and severe, causing damages, destruction and, subsequently, financial losses that propagate to the overall economy. Governments and institutions have already started to take actions to limit greenhouse gases and to encourage the transition to a low-carbon economy. However, this passage must be handled with care. The adaptation and mitigation policies should guide society towards a smooth restructuring of market's functioning. This is so to avoid the repercussions of a late and abrupt transition, which instead could potentially lead to a substantial repricing of climate-related risks. Determining the impacts of climate risk on the financial system is a pressing and prominent issue.

Climate risks are recognized as potential drivers for structural changes [14]. From a market risk perspective, climate factors can significantly affect the value of financial assets.

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They might reshape or disclose new information about upcoming economic circumstances and the price of real or financial assets. This would lead to negative price shocks and an increase in volatility for traded assets. Climate change may even cause a disruption in asset correlations, which might potentially reduce the efficiency of hedges and make it more difficult for banks to properly manage their risks. However, the possibility of unanticipated price shifts may be diminished if climate risk has already been factored in [4]. Changes resulting from the transition might affect borrowing costs and cause a sudden revaluation of financial assets. For example, market investors could reward borrowers who they think will be robust throughout, or may benefit from, the shift away from a carbon-intensive economy. They may even raise the risk premiums for borrowers with high carbon footprints [4]. Despite the uncertainty of the outcomes, it is clear that in the future there will be a mixture of physical and transition risks.

Until now, it is not clear the likelihood, the methodology or the degree at which markets consider the climate as a variable in the pricing of financial assets [4]. The lack of granular data to facilitate quantitative research is one of the major obstacles to identifying climate risk. Standard macroeconomic, financial market, and supervisory reporting data must be combined by central banks and regulators with the latest climate information [15]. Prudential supervision is still at an early stage for the inclusion of climate factors in the system. Nevertheless, in recent times, different authorities started to work in that direction, making some significant advances. They are progressively elaborating new methods and instruments to evaluate the financial risk of climate change, both for physical and transition risk [14].

There are initiatives that are trying to tackle relevant data gap issues and provide reliable disclosures to help the identification of climate risk as financial risks. Notwithstanding these efforts, taxonomies are yet incomplete. In the future, it will be necessary to have climate-data availability and to develop efficient market methods to evaluate the forward-looking obligations. To enhance the results of the assessments, there must be more concern about the second-round effects and the possibility of non-linearities during the modelling phase. Furthermore, it should be studied more in depth also the link between the risks and the financial system, in particular identifying all relevant physical touchpoints; the measures adopted by financial and non-financial corporates; the mitigation of risks in lending with collateral and in insurances; the interaction among acute and chronic physical risk drivers [8].

The existing research develops mostly on the effects of climate risk on the credit risk side, while the market risk is more neglected. The literature puts emphasis on specific market consequences, macroeconomic top-down analyses or sectors of particular economies. Financial markets are trying to rapidly incorporate climate-related risks, but this has not yet resulted in a materially significant difference, leaving room for future substantial repricing. Numerous researchers have looked into the issue of a carbon premium in financial markets, but the evidence is at best conflicting as to whether or not climate risk is completely factored into either the physical risks or the transition risks side [8]. The effects of climate-related risks on the fixed income market are examined only by a limited number of publications. Nonetheless, it would be beneficial to concentrate on the climate influence on bond prices to foresee the impact of climate policies on financial markets and financial stability as well and how they interact. In the literature, a particular field for studying the implications of physical risks in the market is the one related to municipal bonds. This is because municipalities, differently from firms, cannot relocate in case of increasing climate change outcomes. Therefore, it is expected that the debt of those municipalities particularly exposed to physical risks, such as the rise of sea levels, is traded at a significant discount due to the higher riskiness. In this regard, relevant research was made in [20], where it is demonstrated that, when issued, the municipal bond yields are greater for those US counties that have a higher exposure to sea level rise. His findings on municipal bonds suggest that, actually, the market prices climate risk for long-term securities. The effects are however small and, in the short term, there are no significant differences in municipal bonds exposed to climate risk. In a related work [12], the authors find that the effect of sea level rise on the yields is equivalent to a 3-8% reduction in the present value of the long-run cash flows of the local government. The differences in the yields are small but statistically significant. Analysis by BlackRock Investment Institute [10], on the other side, implies that investors in general do not require a climate risk premium for municipal bonds.

Although some of the theoretical mechanisms through which physical and transition risks may influence financial stability are already illustrated by the literature, more research is needed. Given the increasing society's awareness, we develop the idea of assessing the impacts of climate-related risk in the broad financial framework of the fixed income market. Being climate risk a possible cause of structural change as it drives the sentiment of consumers, investors and policymakers, it can reshape all the fundamental variables at the basis of the economy. Due to the forward-looking nature of climate change, we expect that the long-term expectations on interest rates will provide some proof of this connection. In particular, the purpose of this paper is to identify a theoretical model able to introduce a climate factor inside the term structure of interest rates, coherent with real climate change scenarios and able to be calibrated with a forward-looking perspective. Hence we propose a new methodology and apply it to the climate version of the two-factor Vasicek model, where one of the factors corresponds to a climate change variable. Although it is unusual to use a non-financial item inside the model, this element of novelty enables the estimation and measurement of climate risk in interest rates. In our specific case, we will examine the greenhouse gas emissions and temperature rise. Since their consequences will be more pronounced in the long term, an analysis based only on past information would not satisfy the need for a forward-looking view. Following this reasoning, we avail of scenarios that estimate the evolution of climate under different underlying hypotheses to evaluate the future influences. To incorporate it inside the yield curve, the BCC approach suggests to build a set of benchmark curves that reflect the variation of the zero-coupon rates due to climate risk. The distortion is introduced only at some fixed points, therefore, to spread the effect all over the yield curve, we revert the normal process of bootstrapping to find the new implied rates. Our model can then be finally calibrated on each benchmark curve to find the respective parameters.

The paper is organized as follows. In section 2 we describe the BCC approach finalized to recover the benchmark curves and point out their role in the calibration process. Next, section 3, is devoted to the application of the BCC methodology to the climate-Vasicek model, where one of the factors is a climate change variable. Section 4 details the implementation of the model in the EU market and discusses the results obtained. Finally, section 5 draws a number of conclusive points.

2 The Benchmark Curves Calibration approach: climate scenario-based analysis and reverse bootstrapping

In this paper, we introduce a new methodology to assess the impact of climate change on market risk: the Benchmark Curves Calibration approach (BCC for short). More precisely, by adopting a forward-looking approach, we focus on the possible movements of the term structure of interest rates under certain pre-specified climate scenarios. This is done to emphasize the long-term effects that climate risk has, although it is just starting being factored in the market. We rely on the latest IPCC's scenarios [18] that forecast the future global GHG emissions and consequent mean temperatures. The predicted variations in these two variables are used to distort the yield curve, which is recovered following the multi-curve setting [2]. The BCC approach employs them in conjunction with their correlation with the time series of quoted interest rates. Since the distortions are only introduced at some pre-defined scenario-dependent points, we propagate the effects along the yield curve by reverting the process of bootstrapping. This procedure allows us to retrieve the so-defined benchmark curves that represent the curves on which the calibration of a climate model is performed. The whole procedure of the BCC approach is described in detail in the following.

2.1 Climate scenarios

Central banks and supervisors employ scenario analysis as an instrument to evaluate the possible future impacts of climate change on the macroeconomy, financial system and reliability of financial firms [14]. It is not a simple task to measure those impacts. The uncertainty of the path of climate change makes it more involved, together with the complex mechanism of transmission channels and the right integration of primary and secondary effects. The scenario analysis tool is however flexible enough to account for the forward-looking feature of the climate change risks. It offers a methodological approach to structure assumptions about many possible futures in order to investigate potential dangers [14]. To underline the impact of climate threats on the financial system and institutions, the analysis requires speculative but realistic scenarios [8].

Among practitioners, climate scenario construction commonly relies on the works conducted by the Intergovernmental Panel on Climate Change (IPCC), which discloses periodic scientific assessments on climate change, its effects and risks, and offers possible alternatives for adaptation and mitigation. Recently, it was published the Sixth Assessment Report (AR6), with the first part dealing with the last physical understanding of the climate system and climate change. The work provides estimates about future paths presenting different scenarios that we will use to understand the possible consequences of climate change on interest rates. Scenarios picture potential sets of decisions taken by humanity, without any given likelihood if some actions are more probable than others. Inputs of simulations are future concentrations or anthropogenic emissions of radiatively active substances [17]. The scenarios studied vary considering socio-economic and policy responses that may act against emissions and the removal of CO_2 from the atmosphere or, on the contrary, that no limits are introduced and the emissions may even double from current levels [18]. Nowadays, IPCC's scenarios are a subset of the Shared Socioeconomic Pathways (SSP) framework, which can be understood as future emissions and concentration scenarios that depend on the combination of socio-economic development pathways with assumptions on climate change mitigation [6].

The past IPCC reports highlighted already the near-linear relationship between cumulative carbon emissions and global mean warming. This finding suggests that continuous emission of CO_2 will bring on additional warming and changes in all parts of the environmental framework, notwithstanding any particular situation or pathway. Furthermore, the same amounts of cumulative CO_2 emissions could yield slightly different levels of warming over time. For example, quick emissions followed by drastic cuts and likely net negative emissions cause higher maximum warming and faster warming rate compared to the same cumulative CO_2 emissions spread over a longer period [6].

The original report [6] presents five illustrative SSP scenarios and, together, they represent a range of plausible trends, societal and climatic futures. They can be detailed as follows:

- **SSP1-1.9** (*Paris Agreement goal*): it presumes very strong mitigation actions in line with the aim of the Paris Agreement. Therefore, warming reaches approximately 1.5C above 1850-1900 levels in 2100 after a slight overshoot. As regards emissions, the projection (see Figure 1) shows that the implied net zero CO_2 is reached around the middle of the century.

- **SSP1-2.6** (*sustainable pathway*): it entails stringent air-quality mitigation policy, which leads to a fast decrease in particle emissions. In fact, it assumes that the emissions of CO_2 reach the implied net zero in the second half of the century. Warming stays below 2.0C relative to the pre-industrial level.

- **SSP2-4.5** (*middle-of-the-road*): until the middle of the 21st century, CO_2 emissions remain almost the same as today's level. It stands out from a pure 'no-additional-climate-policy' reference scenario, reaching 2.7C of warming by 2100.

- **SSP3-7.0** (*regional rivalry*): it indicates slow improvements since the level of pollutant emissions keeps staying at the current pace. Therefore, CO_2 emissions roughly double from current levels by the end of the century. The scenario assumes reduced air-pollution control and hence higher aerosol emissions.

- **SSP5-8.5** (*fossil fuel-rich development*): this is, with respect to the others, the worst-case scenario. It assumes no additional climate policy, where CO_2 emissions almost double already by 2050 and temperature increases by approximately 4.0C relative to the pre-industrial level by the end of the 21st century.

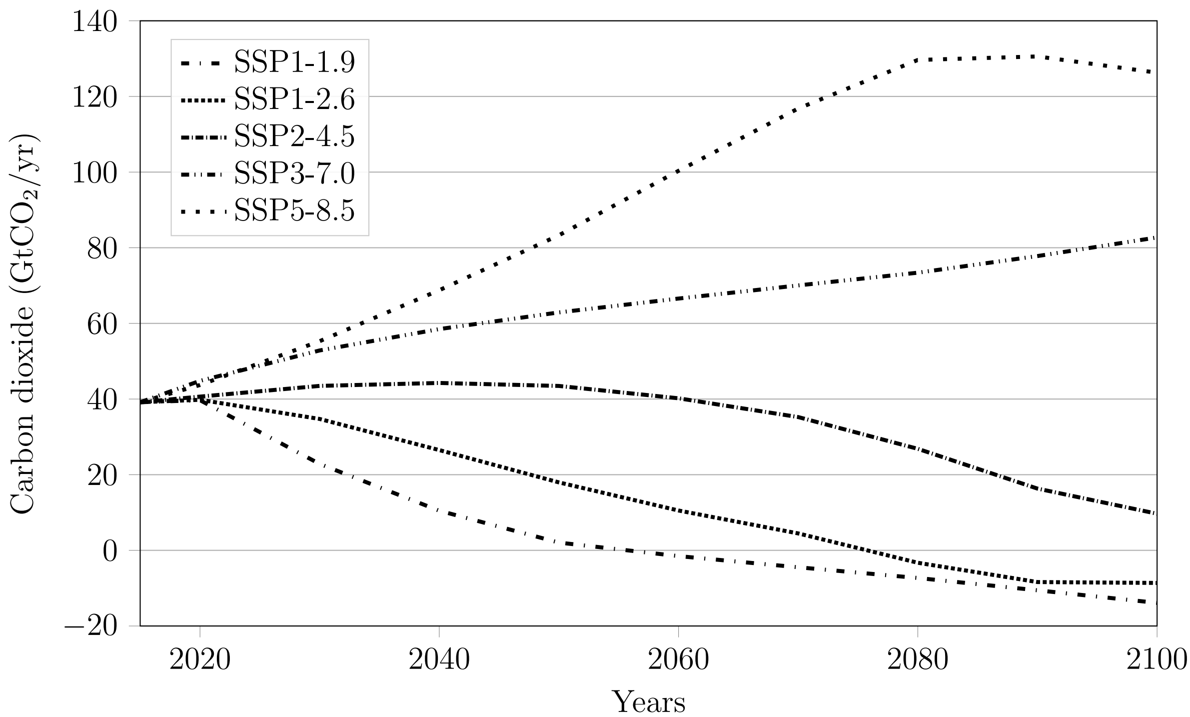

Figure 1: Future annual anthropogenic CO_2 global emissions scenarios. Adapted from Figure SPM.4, Summary for Policy Makers (IPCC 2021).

In Figure 1, the annual global anthropogenic emissions of CO_2 from all sectors for each scenario are plotted. The period considered goes from 2015 to 2100. In the lower part of the graph emerges the scenarios that imply a decrease in emissions, with SSP1-1.9 and SSP1-2.6 reaching even negative values after the middle of the century. In the upper part, instead, we find the worst-case scenarios (SSP3-7.0 and SSP5-8.5) that entail no climate policy mitigation for the abatement of CO_2 in the future years. Emissions are expressed in terms of Giga tonnes of carbon dioxide ($GtCO_2$). In Table 1, we report instead the temperature rise in C for each SSP scenario. The increments are estimated for three periods, starting in 2021, 2041 and 2081. They represent the effects in the near term, mid-term and long term, respectively. In this way, it is possible to measure the rise in temperature until the end of the century at different middle points. The changes are expressed using two reference periods: the pre-industrial one, which goes from 1850 to 1900, and a more recent one that considers the global average mean from 1995 to 2014 as basis. The estimates represent the central value of a confidence interval at 95%.

Table 1: Scenarios of global surface average temperature change. Adapted from Table 4.5, section 4 (IPCC 2021).

TEMPERATURE SCENARIOS (C)						
Time Period	**Basis**	**SSP1-1.9**	**SSP1-2.6**	**SSP2-4.5**	**SSP3-7.0**	**SSP5-8.5**
2021-2040	1995 - 2014	0.6	0.6	0.7	0.7	0.8
	1850 - 1900	1.5	1.5	1.5	1.5	1.6
2041-2060	1995 - 2014	0.7	0.9	1.1	1.3	1.5
	1850 - 1900	1.6	1.7	2.0	2.1	2.4
2081-2100	1995 - 2014	0.6	0.9	1.8	2.8	3.5
	1850 - 1900	1.4	1.8	2.7	3.6	4.4

2.2 Yield curve construction under the modern multiple-curve framework

The yield curve is a fundamental object in the market due to its relevance and it is at the basis of this research. To derive it, the preferred market method is the exact-fit [2]. The purpose is to exactly reprice N selected market instruments using the yield curve fixed on a time grid, i.e. a predetermined vector of dates named pillars, at which the bootstrapping procedure returns a value. In this case, the technique is called bootstrap since the implementation is recursive, meaning that the curve is constructed step-by-step using the pricing formula of one instrument per time for increasing maturities. Beyond the values fixed at the i-th pillars, through interpolation, it is possible to recover the intermediate values of the curve [[2], [13]].

The global subprime credit crunch crisis started in 2007 and the Eurozone sovereign debt crisis of 2009-2012, which triggered a new regime of low and even negative rates, have irreversibly modified the functioning of the financial markets as they were previously known. Quoted instrument prices started to show that the relationship between Libor rates with different maturities should be modelled separately and, for the first time, the spread between Libor rates and the swap rates on the Overnight Indexed Swaps became significant [13]. This does not mean that before the Basis Swaps spreads were zero, they were just sufficiently small to be safely neglected [1]. Therefore, in the post-crisis setting, it is recognized that the *counterparty risk*, i.e. the risk of failure of a counterparty to fulfil its obligations, and the *funding risk*, i.e. when the lack of liquidity in the market causes a risk of excessive costs of funding a position, affect the interbank rates. This effect spreads consequently all over the fixed-income market since the underlying rates of most fixed-income instruments are exactly those kind of rates [1]. These new aspects contributed to the diffusion of the multiple-curve framework. An essential point to emphasize in this setting is the requirement of *homogeneity*: interest rate derivatives with a given underlying rate tenor must be priced and hedged using vanilla interest rate market instruments with the same underlying. In this context, it is fundamental to coherently construct and use the discounting curve. By no-arbitrage, the present value of any future cash flow must be unique and, consequently, the discounting must be the same for all the curves. Building the correct discounting curve implies being consistent with the funding of the selected quoted bootstrapping instruments, which are mainly traded OTC. Since they are traded between counterparties under bilateral collateral agreements, or with central counterparties, with daily margination at the overnight collateral rate, the coherent instruments for the discounting curve construction are the Overnight Index Swaps [2]. This is the so-called *funding* requirement.

To sum up, we set the standard procedure followed by the market to develop the multiple interest rate yield curves [2]:

1. Choose the appropriate funding rates, select the corresponding market instruments and build the unique discounting curve using the usual single-curve approach. It will be derived from the OIS.

2. Pick a set of homogeneous vanilla interest rate instruments in their underlying rate with increasing maturities, one set for each different tenor considered.

3. Construct the FRA curves for each tenor using the selected instruments' market values and their bootstrapping rules, using the unique discounting curve obtained in point 1.

4. Recover the relevant FRA rates and the corresponding cash flows from the FRA curve, always observing the homogeneity principle.

5. Compute the relevant discount factors from the discounting curve with the proper funding features.

6. Sum the discounted cash flows to obtain the prices.

2.3 Climate factor distortion

After recovering the relevant market yield curve, we follow the hypothesis under the IPCC's scenarios to build a method that allows us to analyse the future possible impacts on it relying on climate change variables. We know that, until recent years, those effects were not considered a major issue. This is because climate risk is perceived in a more disruptive way in the long-term, while in the short-term it does not rise the same concerns. In fact, the worst impacts have not yet happened and will not happen even in the next few years. Having said this, it is clear that we have to work coherently with the forward-looking nature of climate risk. At first, we can start relying on historical data to realistically size this effect. Notice that, using past information, we should however obtain a slightly optimistical and biased view of what will happen in the future. We use Pearson's ρ

correlation between the time series of the chosen interest rate instruments and the climate change variable. In this way, we recover the dependence among the two variables that will govern the shift that we introduce. Moreover, we compute the percentage change variation of the climate variable under the assumption of a specific scenario. To modify the outcome of our original market curve, we thus introduce the shift at the appropriate zero-coupon rate. More formally, we have

$$\Delta R(t,T_i) = R(t,T_i) + R(t,T_i) \times \rho_{i,k} \times \Delta scenario_{k,j}, \tag{1}$$

for $R(t,T_i)$ defined as the market zero rate with maturity T_i at time t, $\rho_{i,k}$ as the Pearson correlation between the instrument with maturity T_i and the climate change variable k, and $\Delta scenario_{k,j}$ as the variation in k due to the selected scenario j.

2.4 Reverse bootstrapping

Since we introduce the fixed variations only at specific points in time, we have to propagate the changes to all the rest of the curve in a proper manner. Accounting for the fact that, today, the rates are fixed, we can modify only those that are quoted in the future, ideally from tomorrow. We thus should restore the yield curve by keeping some points fixed (the values at today and the exogenous shifts), which must be linked in the smoothest possible manner. We could rely on some interpolation scheme, however, connecting just few points in a long period is a huge simplification. Therefore, to absorb the distortions over time, we use the idea of reverse bootstrapping, meaning that we invert the usual bootstrapping procedure such that we obtain the previous value of the zero coupon bond instead of the following one. In this manner, we can modify all the nodes depending on the shifts we introduce in a more reliable fashion.
The distortion is introduced into the zero-rates so, to spread the effects, we have to compute them for each different scenario as follows

$$\Delta P(t,T_i) = \exp\Big\{ -\Delta R(t,T_i)\tau(t,T_i) \Big\}.$$

Differently from the bootstrapping procedure, we start the process from the last (future) value and turn back until the most recent value. Following [2], we retrieve the inverted formulas of the instruments used to build the yield curve to recover the previous value, i.e. the value at time T_{i-1}. Therefore, we obtain for Depos, FRA and IRS rates, respectively, the distorted zero-coupon bond prices as:

$$\begin{aligned}
\Delta P(t,T_i) &= \frac{1}{1+\Delta R^{Depo}(t;T_i)\tau_L(t,T_i)}, && (2)\\
\Delta P(t,T_{i-1}) &= \Delta P(t,T_i)(1+\Delta R^{FRA}(t,\boldsymbol{T_i})\tau_L(T_{i-1},T_i)), && (3)\\
\Delta P(t,T_{i-1}) &= \left[\Delta R^{IRS}(t;T_i)A_c(t;T_i) - \sum_{\alpha=1}^{i-1} P_c(t,T_\alpha)F(t;T_{\alpha-1},T_\alpha)\tau_L(T_{\alpha-1},T_\alpha) + P_c(t,T_i)\right]\\
&\quad \times\frac{\Delta P(t,T_i)}{P_c(t,T_i)}.
\end{aligned}$$

With the above formulas, we can set an optimization problem subject to some constraints: the starting point $P(t,t) = 1$ and the distorted zero-coupon bonds. Moreover, we have to make an additional assumption. The forward rates of IRS do not change due to the distortions. This is to assure that the numerical solution propagates the effects of the climate change variables in time, otherwise they would be absorbed only in the years near the introduction of the distortion. Once the optimization is done, we can retrieve the rates and interpolate them to find the entire benchmark curve for each scenario.

2.5 Calibration: least squares optimization

Given a short-rate model, the idea of calibration is to compare the market zero-coupon rates to the model zero-coupon rates. Therefore, we first recover the corresponding zero rates from our benchmark curves, which we simply denote as $R(t, T_i)$. We then retrieve the set of model's parameters Θ by minimizing the sum of the squared residuals between observed and predicted values. The objective function corresponds then to

$$\min_{\Theta} \sum_{i=1}^{n} \Big(R(t, T_i) - \bar{R}(t, T_i) \Big)^2 ,$$

where the zero rate from the model $\bar{R}(t, T_i)$ is given by the logarithm of the zero-coupon bond. Usually, the model's parameters are calibrated on the time series of the interest rates and then used to predict the short rates. However, we slightly modify this procedure due to our particular setting, while preserving the concept of calibration. In fact, we cannot rely on past data, since the historical parameters will not capture the distortions that we introduced in the benchmark curves and are not able to replicate them. We thus have to perform a calibration on the future values of the zero rates to recover the right parameters for each different scenario. This is exactly the reason why we derived the benchmark curves. In addition, to retrieve the value of the model's parameters, we have to deal first with other unknown factors. As a matter of fact, the short rate and the climate change factors y and c are stochastic but, since we base our calibration on the future, we have to randomly simulate their values and use them as if they were known. Being a random estimation, to obtain reliable results we have to simulate the values a large number of times, n, and find the model's parameters as the mean of all the parameters recovered in each simulation. We thus use a similar idea to that of the Monte Carlo method, i.e that repeated random sampling allows recovering good numerical results on average.

3 The BCC approach at work: an affine climate-factor model

The calibration phase of the BCC approach is fundamental since entails the selection of an appropriate short rate model. Our aim is to retrieve the parameters of a model able to replicate the shape of the benchmark curves. This method relies on the assumption that interest rates have a component linked to climate change, we can therefore express the short-term interest rate $r(t)$ as a linear combination of two variables:

$$r(t) = c(t) + y(t), \tag{4}$$

where the factor representing the climate change is denoted by c, while y denotes all the other components that define interest rates.
The above expression allows us to rely on affine term structure models, which conveniently link the logarithm of bond prices to spot rates through a linear function. In their simplest version, i.e. the one-factor case, the zero coupon price can be indeed expressed

by considering a class of compatible models (F, μ, σ) with F defined as an exponential affine equation [11], therefore:

$$F(t,T;x) = \exp\big\{A(t,T) - B(t,T)x\big\}, \tag{5}$$

for which A and B are C^1 functions on $[0,\infty)$. By computing the partial derivatives and solving the term structure equation, we obtain

$$A'(t,T) - (1 + B'(t,T))r - \mu(t,r)B(t,T) + \frac{1}{2}\sigma^2(t,r)B^2(t,T) = 0. \tag{6}$$

The usual boundary value for zero coupon bonds $F(T,r;T) = 1$ implies that

$$\begin{cases} A(T,T) = 0 \\ B(T,T) = 0 \end{cases}$$

The existence of functions A and B that solve equation (6) is usually guaranteed by choosing μ and σ^2 as affine functions of $r(t)$ [11]. By doing so, equation (6) becomes a separable differential equation for the unknown functions A and B. Assuming that μ and σ have the following form

$$\begin{cases} \mu(t,r) = \alpha(t)r(t) + \beta(t) \\ \sigma(t,r) = \sqrt{\gamma(t)r(t) + \delta(t)} \end{cases}$$

the functions A and B satisfy the systems

$$\begin{cases} B'(t,T) + \alpha(t)B(t,T) - \frac{1}{2}\gamma(t)B^2(t,T) = -1 \\ B(T,T) = 0 \end{cases} \tag{7}$$

$$\begin{cases} A'(t,T) = \beta(t)B(t,T) - \frac{1}{2}\delta(t)B^2(t,T) \\ A(T,T) = 0 \end{cases} \tag{8}$$

Notice that equation (7) is a Riccati equation that does not involve A. However, we are interested in the multi-dimensional setting of affine models, especially in the two-factor case. Therefore, the two variables that describe the short rate as presented in equation (4) must be correlated state variables that each follows its own stochastic differential equation, expressed in general terms as

$$\begin{aligned} dc(t) &= \nu(t;c)dt + \rho(t;c,y)d\hat{W}_c(t), \\ dy(t) &= \nu(t;y)dt + \rho(t;y,c)d\hat{W}_y(t), \end{aligned}$$

where the diffusion coefficient entails the covariance between the sources of uncertainty. Applying the Itô's theorem to retrieve the differential dynamics of the zero-coupon bond price, we recover

$$\begin{aligned} dP(t,T;c,y) \; = \; & \left(\frac{\partial P}{\partial t} + \frac{\partial P}{\partial c}\nu(t;c) + \frac{\partial P}{\partial y}\nu(t;y) + \frac{1}{2}\frac{\partial P}{\partial c \partial y}\rho(t;c,y) + \frac{1}{2}\frac{\partial P}{\partial y \partial c}\rho(t;y,c)\right)dt \\ & + \frac{\partial P}{\partial c}\rho(t;c,y)d\hat{W}_c(t) + \frac{\partial P}{\partial y}\rho(t;y,c)d\hat{W}_y(t). \end{aligned}$$

We obtain then the partial differential equation under the risk neutral measure as:

$$\begin{aligned} & \frac{\partial P}{\partial t} + \Big(\nu(t;c) - \lambda_c\rho(t;c,y)\Big)\frac{\partial P}{\partial c} + \Big(\nu(t;y) - \lambda_y\rho(t;y,c)\Big)\frac{\partial P}{\partial y} + \frac{1}{2}\frac{\partial P}{\partial c \partial y}\rho(t;c,y) + \\ & + \frac{1}{2}\frac{\partial P}{\partial y \partial c}\rho(t;y,c) - cP(t,T) - yP(t,T) = 0, \end{aligned}$$

where λ_c and λ_y represent the market price of risk.
The zero coupon bond is then a function of two underlying risk factors, one of which represents the climate change risk, and can be expressed as an exponential affine expression:

$$P(\tau; c, y) = e^{A(\tau) - B_c(\tau)c - B_y(\tau)y}, \tag{9}$$

where we use τ to indicate the time to maturity, i.e. $T - t$, and the factors y and c are written without their dependence on time just to ease the notation. This is the generalization of an affine climate-factor model, which can then be adapted to a specific short rate model.

3.1 The Climate-Vasicek model

In order to further specify the model, we express it as a Climate-Vasicek model, i.e. as a two-factor affine term structure with a climate change component using the dynamics of the Vasicek short-term rate model. In fact, it turns out that it satisfies the necessary affinity conditions [11] and can be easily adapted to a climate version.
In general terms, the differential equation consists in an Ornstein-Uhlenbeck process with constant coefficients [21] for a proper choice of the market price of risk, that is

$$dr(t) = \kappa(\bar{\theta} - r(t))dt + \sigma dW(t), \quad r(0) = r_0, \tag{10}$$

where:

- $\bar{\theta}$ is the long term mean level, which means that all the future trajectories of the rate will evolve around that term;
- κ is the speed of reversion, i.e. the velocity of the long term mean level that will be reached, and it should be positive;
- σ is the instantaneous volatility that measures the amplitude of randomness introduced in the model.

The choice to employ this model is due to its well-known characteristics. The time-homogeneous structure of the model allows only for constant coefficients, which is convenient when retrieving analytically the price of bonds even in the two-dimensional case. However, this could be also seen as a disadvantage since few constant parameters cannot account for all the features and shapes of yield curves. In this model, the rate $r(t)$ can be negative with positive probability. Although in recent times the interest rates turned positive almost everywhere, until no long ago the majority of the economies were under the NIRP regime. The short rate is also mean reverting. It means that the expected rate will tend to the value of $\bar{\theta}$, which is indeed the long term average rate, for t that approaches infinity. Another interesting quantity to consider is the long term variance $\sigma^2/(2\kappa)$, which comes from considering the variance as $t \to \infty$. If we consider the limit for $T \to \infty$ of the zero-coupon rate, we have

$$R(t, \infty) = \lim_{T \to \infty} R(t, T) = \bar{\theta} - \frac{\sigma^2}{2\kappa^2}.$$

It indicates that, as the time to maturity increases up to infinity, the yield curve will converge to (or eventually deviate from) the value of the long-term mean influenced by the volatility and speed of reversions.
Following [5], we can compute the partial derivatives of equation (9) expressing the drift

and the diffusion coefficients of the climate two-factor case considering the Vasicek dynamic:

$$\mu_c = \kappa_c(\bar{\theta}_c - c(t)), \quad \rho_{cy} = \sigma_{cy}$$

$$\mu_y = \kappa_y(\bar{\theta}_y - y(t)), \quad \rho_{yc} = \sigma_{yc}$$

where $\bar{\theta}_i = \theta_i - \frac{\sigma_i \lambda_i}{\kappa_i}$ for $i = c, y$. Putting all together in (9), we obtain the following partial differential equation:

$$\begin{aligned}
&-A'(\tau) + \frac{\sigma_{cy}}{2} B_c(\tau) B_y(\tau) + \frac{\sigma_{yc}}{2} B_y(\tau) B_c(\tau) - \kappa_c \bar{\theta}_c B_c(\tau) - \kappa_y \bar{\theta}_y B_y(\tau) + \\
&-\big(1 - B_c'(\tau) - \kappa_c B_c(\tau)\big)c - \big(1 - B_y'(\tau) - \kappa_y B_y(\tau)\big)y = 0.
\end{aligned}$$

We split the above equation into a series of ordinary differential equations to be able to solve them analytically.

$$\begin{cases} B_c'(\tau) + \kappa_c B_c(\tau) = 1 \\ B_c(0) = 0 \end{cases}$$

leads us to recover

$$B_c(\tau) = \frac{1}{\kappa_c}(1 - e^{-\kappa_c \tau}). \tag{11}$$

Following the same reasoning, we obtain

$$B_y(\tau) = \frac{1}{\kappa_y}(1 - e^{-\kappa_y \tau}). \tag{12}$$

Then we are left to solve the following system, i.e.

$$\begin{cases} -A'(\tau) + \frac{\sigma_{cy}}{2} B_c(\tau) B_y(\tau) + \frac{\sigma_{yc}}{2} B_y(\tau) B_c(\tau) - \kappa_c \bar{\theta}_c B_c(\tau) - \kappa_y \bar{\theta}_y B_y(\tau) = 0 \\ A(0) = 0 \end{cases}$$

In this case, we integrate the expression to find the function $A(\tau)$. We can recover the closed-form expression for $A(\tau)$, after rearranging it in a convenient order, as

$$\begin{aligned}
A(\tau) \quad = \quad & \frac{\gamma_c(B_c(\tau) - \tau)}{\kappa_c^2} - \frac{\sigma_c^2 B_c^2(\tau)}{4\kappa_c} + \frac{\gamma_y(B_y(\tau) - \tau)}{\kappa_y^2} - \frac{\sigma_y^2 B_y^2(\tau)}{4\kappa_y} \\
& + \frac{\sigma_{cy}}{2\kappa_c \kappa_y}\left(\tau - B_c(\tau) - B_y(\tau) + \frac{1}{\kappa_c + \kappa_y}\big(1 - e^{-(\kappa_c + \kappa_y)\tau}\big)\right),
\end{aligned}$$

where

$$\gamma_c = \kappa_c^2\left(\theta_c - \frac{\sigma_c \lambda_c}{\kappa_c}\right) - \frac{\sigma_c^2}{2},$$

$$\gamma_y = \kappa_y^2\left(\theta_y - \frac{\sigma_y \lambda_y}{\kappa_y}\right) - \frac{\sigma_y^2}{2}.$$

Putting all together in equation (10), we obtain the complete analytical version of the Climate Vasicek model with two-factors, one of which represents the climate change risk.

4 Model implementation and results discussion

In this section, we apply the BCC approach in the EUR market. Our aim is to find out and study the effects of each climate scenario disclosed by IPCC on the rates in the Eurozone framework. Therefore, following the methodology described above, we first recover the yield curve quoted in the market under the multi-curve framework. This curve is used as a basis for the construction of five benchmark curves, each one incorporating a future different path of the underlying climate change variation. The distortions at three different points in time are introduced in the original curve by adding an adequate variation to the zero rates, which depends on the scenario selected and on the correlation between interest rates and the climate change variables. To propagate the impact to the entire curve, we reverse the procedure of bootstrapping. After retrieving the benchmark curves, it is time to calibrate the parameters of the Climate Vasicek model by minimizing the squared difference between the original rates and the model rates. At this point, it is possible to evaluate the results and draw interesting conclusions, insightful also from a policy stability point of view.

4.1 Variables selection

4.1.1 Interest rates

We recover from the Datastream-Eikon database the daily time series of the interest rates instruments with 6 months tenor as underlying, which is the relevant one in the Euro market, from January 1, 2015, to December 31, 2021. We opt for this 7-year period because it covers two major interesting events that are related to climate change: the Paris Agreement and the Covid-19 pandemic. The first one is a milestone for tackling climate change by countries around the world and notably dragged the attention of the public to the consequences of no intervention in the short term. The second one, instead, was a disruptive event from many points of view, leading to economic and environmental consequences. It rose awareness about the human impact on nature and the importance to preserve the ecosystem. We expect that these two events influenced the market sentiment towards a more climate-friendly society. Accounting for the liquidity of the instruments, we select the Deposit at 6 months, the FRAs with 6 months expiry (in particular, 1x7, 2x8, 3x9, 4x10, 5x11, 6x12, 12x18) and the IRS (from 2 to 60 years) with fixed leg tenor 1 year and floating leg 6 months. Analysing the different time series, we find missing data and outliers only in the IRS quotes. To preserve the reality of data, we substitute their values with the average mean between the quote of the day before and the day after the outlier value.

4.1.2 Climate change variables

Greenhouse gas emissions. The first climate change variable that we consider is the amount of greenhouse gases' air emissions. Due to its primary role in causing physical and transition risks, it is commonly used as a driver for the construction of future climate scenarios and policy makers act directly on its limitation.

The Eurostat Database publishes quarterly data of air emissions accounts for greenhouse gases by NACE Rev. 2 activity. Data are recovered quarterly for the period from January 1, 2015, to December 31, 2021, expressed in thousand tonnes and as a collection of CO_2 and N_2O, CH_4, HFC, PFC, SF_6 and NF_3 in CO_2 equivalent. The countries considered are the 27 members of the European Union. We select the total emissions of all NACE activities plus those of the households. Since we have quarterly data but privilege daily frequency, we assume that emissions are constant inside each quarter. Hence, each quarter's data is divided by the number of business days in it to obtain the daily emissions. We are aware that this is a strong assumption, emissions are hardly constant over such a long period. Moreover, in this way, we do not account for the change that emerges during weekends or holidays. Despite this, we can still get an idea of the real path of GHG emissions.

Surface mean temperature. Another essential variable in the climate change context is temperature. High and increasing levels of GHG concentration in the atmosphere result in a rise in the global surface temperature, which in turn causes more extreme weather events. For this reason, it is usually used as a proxy for measuring the physical risks of climate change.
The National Oceanic and Atmospheric Administration (NOAA) website provides the access to the National Climatic Data Center's archive of global historical weather data. From that database, we collect the daily average, maximum and minimum temperatures for each meteorological station of the members of the European Union. The time series collected refers again to the period from January 1, 2015, to December 31, 2021. Each state has a different number of observations due to the size of the country, the number of stations and missing values. For these reasons, we follow a common method to find the average temperature in every single country.

To deal with missing mean temperature data, we adopt the following procedure:

1. If both minimum and maximum temperatures are available, we compute the average temperature as the mean between them.

2. If there are still more than 365 missing observations (1 year out of 7 of data) of average temperature for a station, we remove it entirely.

3. If there are missing days of data recording, we add the related values propagating the last valid observation forward to the next available.

The data is then sorted by date in order to compute the average across stations. Once this is done for every country in the dataset, it is possible to obtain a European daily average by taking the weighted mean across each day and country. The weights are given by the surface of the country, which is retrieved from the Eurostat Database. Finally, the weighted daily average temperature of the entire EU is converted from Fahrenheit to Celsius degrees.

Carbon allowances. Carbon allowances are the main tool to limit GHG emissions, based on a cap-and-trade mechanism. We can consider them representative of the assessment of the risk of the transition. As we know, it is a particular market and we will not consider carbon prices as a climate variable in our model. However, we investigate the prices in connection with emissions and interest rates. We expect to find dependence between these variables that can be used to support the effects of climate change that we find out.
We collect daily data about carbon allowance prices in the Euro market from the Datastream-Eikon database. In particular, we recover the EEX EUA spot prices for the period January 1, 2018, and December 31, 2021. Hence, the data refers to the last 3 years of phase 3, including the Covid-19 crisis, and the initial part of phase 4. The market was under particular stress in that period, therefore the information may be biased. For this reason, we have to be careful about the interpretation of the numbers related to such a variable.

4.2 Analysis of the dependences

To investigate the effects of climate change on interest rates, a good starting point is analysing their past relationship expressed as Pearson's ρ. Being this the ratio between the covariance of two variables and the product of their standard deviations, we have first to deal with the multiplicity of interest rates collected. Indeed, for the latter, we have as many time series as the number of instruments we selected. We proceed by taking the average across the time series of the rates. In particular, since the IPCC's scenarios predict values almost every 20 years, we compute three different averages: one using all the instruments with maturity from 6 months to 20 years, one for the quotes with maturity until 40 years and one for the remaining maturities until 60 years.
In Table 2, we report the Pearson correlation coefficients we recover for the three interest rates averages. The correlation between emissions and interest rates is relatively high and positive. This means that when emissions increase, also interest rates increase. Despite this, we should not confuse the idea that correlation means causation. In fact, it is more an indirect effect as we know that emissions have an influence on the economy, which propagates to the financial markets. As expected, the correlation is higher for the longest maturities, even if the difference is small. We interpret this positive trend as a sign of the increasing relevance of climate change as time passes by. As regards the relationship

Table 2: Pearson correlation coefficients between interest rates and emissions and interest rates and mean temperature for the period 2015-2021.

VARIABLES		**EMISSIONS**	**TEMPERATURE**
INTEREST RATES (average)	**6M - 20Y**	0.5147	-0.0810
	20Y - 40Y	0.5319	-0.0679
	40Y - 60Y	0.5332	-0.0665

between interest rates and average temperature, the correlation coefficients are small, very close to zero, and negative. Moreover, they decrease in time. At first, this might look controversial and unexpected. However, we have to account for the characteristics of this particular factor. It is a pure climate variable that cannot be directly controlled by humans. It depends on a variety of elements, among which the most relevant is the quantity of emission concentration in the atmosphere. Another fundamental aspect is the time required to account for a variation. By this, we mean that the average temperature changes over the years in a less evident way with respect to emissions. It takes decades, or more, to account for huge fluctuations in the values. Here we consider the data from only 7 years of observations. Despite the fact that GHG emissions decreased, the positive effects of this reduction will be presumably visible later in the years. Being temperature a natural variable, it is more difficult for the market to account for it. Also, in this case, we talk about an indirect effect of temperature on the financial system. The extreme weather events caused by the increase in mean temperature are those that actually have an influence due to the monetary losses that they produce. However, it is harder to account for floods, fires or heatwaves. For these reasons, we use the temperature as a general variable in our model that synthetises the physical risk of climate change.
To analyse the effects of the two major events identified in the examined time interval, we partition the period under investigation into three sub-periods: the pre-Covid-19 years (2015-2019), the year of Covid-19 (2020) and the post-Covid-19 era (2021). Reducing the

time size is dangerous because it could provide misleading interpretations, in particular with these kinds of variables that are also characterized by a seasonal component.
The linear correlations between GHG emissions and interest rates computed considering the sub-periods show interesting trends (Table 3). For the years immediately after the Paris Agreement, excluding the crisis due to Covid-19, we have a positive correlation of about 0.3 with all the interest rates averages. This is in line with the value obtained when considering the entire period. The same is true for 2020, although the correlation with the 6-month - 20 years average turns negative. While it could be strange at first, we have to remember what happened. The relationship has been reversed because GHG emissions were subject to a huge decrease due to lockdowns and the market was under stress. This was reflected only on the nearest term because it was a temporary shock. In 2021, the correlation turns back to the same level as the pre-Covid period. On the contrary, the other two averages of interest rates exhibit a negative sign now. Emissions, in fact, rose again in 2021, while interest rates stabilized. It was still a year of changes. Consequently, it is hard to interpret if this modification of the correlations' sign represents a crucial moment for the future or if it is just temporary. A negative correlation in the long term could indicate that the current market sentiment towards emissions lowers because their effects, from 2060 onwards, will have already taken place. It is like to say that the issues related to GHG emissions will no longer be a priority because the actions taken today will be effective by then. In Table 4, instead, we display the results of the

Table 3: Pearson correlation coefficients between emissions and interest rates for the entire period and for the three sub-periods.

		EMISSIONS			
		2015-2021	**2015-2019**	**2020**	**2021**
INTEREST RATES (average)	**6M - 20Y**	0.5147	0.3178	-0.1614	0.3304
	20Y - 40Y	0.5319	0.3098	0.3133	-0.1714
	40Y - 60Y	0.5332	0.3049	0.3961	-0.2296

sub-period correlations between interest rates and the average surface temperature. The pre-Covid-19 period and the year of Covid-19 both show a negative correlation for all the interest rates. However, in 2020, the size of the correlation increases notably. As we said before, it was a particular year for the market and for the climate. Despite this, the consequences on average temperature were not so incisive as those in the market. Only interest rates were subject to substantial change during the pandemic. The next year, the correlations change sign, becoming positive and higher. It could be a true indication of the effects of Covid-19. In fact, the pandemic was not only impactful on the market because of the monetary consequences of the various lockdowns, but also on the awareness of society. Consumers and investors started to take more into consideration the effects of climate change. Although the transition risks are already recognized by policymakers, less attention is directly paid to physical risks. This change in the sign of correlation coefficients could indicate a turning point in the market sentiment. People finally realize the negative consequences of the rise in temperature and this awareness is reflected in the market expectations. Nevertheless, this aspect should be investigated further in the next years to confirm the interpretation. As regards allowances, in Table 5, we find out that for the entire period Pearson's ρ is negative and quite high. It indicates that a decrease in interest rates is related to an increase in the price of allowances, meaning

Table 4: Pearson correlation coefficients between mean temperature and interest rates for the entire period and for the three sub-periods.

		TEMPERATURE			
		2015-2021	**2015-2019**	**2020**	**2021**
INTEREST RATES (average)	**6M - 20Y**	-0.0810	-0.1471	-0.2437	0.1060
	20Y - 40Y	-0.0679	-0.1259	-0.4533	0.3090
	40Y - 60Y	-0.0665	-0.1225	-0.4432	0.2574

that it is costly to pollute when interest rates are lower. Accordingly, it has an inverse relationship with respect to the one between interest rates and emissions. If the price of allowances increases, the emissions should decrease and both these effects influence the interest rates in a negative way. The association holds both for the pre-Covid-19 crisis and during it. However, in 2021 the correlations' sign change. It could be, coherently with the interpretation we gave for GHG emissions, that the market sentiment changed after the pandemic due to the rise in awareness. From January 2021, the fourth phase has started, introducing new mechanisms, influences and prices on the market. Therefore, it could be that the correlations are biased by the consequences of this new phase.

Table 5: Pearson correlation coefficients between allowances and interest rates for the entire period and for the three sub-periods.

		ALLOWANCES			
		2018-2021	**2018-2019**	**2020**	**2021**
INTEREST RATES (average)	**6M - 20Y**	-0.4716	-0.7451	-0.5903	0.6381
	20Y - 40Y	-0.4594	-0.7273	-0.1451	0.1489
	40Y - 60Y	-0.4653	-0.7093	-0.0066	-0.0139

Table 6: Pearson correlation coefficients between temperature, emissions and allowances for the entire period and for the three sub-periods.

	2015-2021	**2015-2019**	**2020**	**2021**
TEMPERATURE EMISSIONS	-0.6593	-0.8017	-0.6435	-0.6890
	2018-2021	**2018-2019**	**2020**	**2021**
EMISSIONS ALLOWANCES	-0.0813	-0.5207	0.3043	0.2964

Lastly, in Table 6, we examine the linear correlations among our climate change variables. They are meaningful to explain the inner connections. Contrary to our expectations, the temperature and the emissions are negatively related. This is probably because there is a time lag between these two variables. We know that an increase in emissions concentration causes an increase in the surface mean temperature. However, their reduction does not imply directly a decrease in temperature or, at least, not immediately. It takes time. This is clear from the SSP1-1.9 scenario where, although emissions become even negative, the temperatures still rise until a point at which they slowly start decreasing. Therefore, the correlation coefficient is not a trustworthy measure of their relation. They are linked by a causation effect. Additionally, the correlation between emissions and allowances is also counter-intuitive. For the entire period, it is almost 0, which indicates a weak connection. Nonetheless, if we split it into intervals, the reasoning becomes clearer: there is a sort of compensation in the correlations during the analysed period. Before Covid-19, the coefficient is negative and quite high. It correctly indicates that an increase in the price of allowances corresponds to a decrease in GHG emissions. This is reasonable. From 2020 a weird effect starts, causing a positive correlation. The price of allowances in 2020 was subject to a huge decrease but it sharply increased again thanks to the MSR mechanism. Hence, part for a small period, the prices stayed stable. The same was not true for emissions, which started to increase again after a long time and at a higher level than before. This mismatch of the effects modified the connection between the two variables.

4.3 Bootstrapping market yield curves

We construct now the market curve, which is needed as a basis for the creation of the modified curves under climate scenarios. In particular, we choose as reference day July 8, 2022, once the market closed, collecting all the quotes needed from the brokers ICAP and KLIEM through the Refinitiv Eikon platform. The construction of the curves is done using the QuantLib framework, which is an open source library used also in the real world to model, trade and deal with risk management. We use the implementation on Python and follow [3] as a reference for its correct usage. We also stick to the suggestions in [2] as regards the selection of the instruments.
From now on, we will mention July 8, 2022, as today (t_0), while July 12, 2022, as spot ($t_0 + 2$ business days).

OIS discounting curve. First of all, we build the discounting curve that is needed to recover all the other rates in the multi-curve framework. Remember that since EONIA was substituted and is no longer available from the beginning of 2022, we use the instruments on the current overnight rate quoted in the Euro market, i.e. €STR.
To construct the OIS curve starting today, we use the three first quoted Depos in the very short term. In fact, we select those with maturity Overnight (ON), Tomorrow night (TN) and Spot night (SN) that have as settlement rule, respectively, today, tomorrow and spot. Notice that there is a small inconsistency in this first part of the curve because we are using instruments written directly on the Euribor and not on the overnight rate. We then choose the OIS written on €STR with maturity between one week and 60 years. In addition, we consider the forward OIS on ECB announcement dates, useful for reflecting the monetary policy decisions of the Central bank. In fact, we highlight that the month of July 2022 in the Euro market is a particular moment due to the high inflation that is hitting the entire world. To contrast this sharp increase, it is expected that the ECB will rise, for the first time in more than a decade, the interest rates by the end of the month. Therefore, at the chosen reference date, some of these expectations are already reflected in interest rates even if the official rise has not yet happened. In Appendix A, in table 17, are reported all the instruments used for the construction of the OIS discounting curve with the details about the start and maturity date of each instrument. Once we have selected the bootstrapping instruments, we can derive the zero coupon bonds from Depos and OIS.

Euribor 6 months yield curve. We can proceed with the construction of the Euribor 6 months yield curve. There are different possible choices for the selection of the bootstrapping instruments and the technicalities of the procedure. It is rather subjective since there is no unique rule. It depends on the market, the availability of data and the curve construction's aim. In Appendix A, table 18, we report all the instruments we use for its construction. The first instrument we select is the Depo with a maturity of 6 months. Next, we include the 7 quoted FRAs starting in 1 month until 12 months, with the Euribor 6 months as underlying. The term structure is then completed by using the IRS on 6 months tenor from 2 years to 60 years. Notice that, differently from the discounting curve, we evaluate the curve at the spot date. Despite this, it is enough to discount the entire curve using the spot discount factor to obtain the curve with the reference date today.
We can recover the FRA curve, the zero-coupon bond curve and, consequently, the zero-coupon curve. The last one is of major interest since zero rates are needed for the implementation and calibration of the two-factor Vasicek model.

4.4 Climate factor distortion

4.4.1 Emission scenarios

Following the procedure presented before, we have first to find two values: the correlation between interest rates and emissions and the future variation forecasted by the IPCC's scenarios. For the first passage, we use the daily time series of GHG emissions in Europe that we discussed in combination with the time series of IRS with 20, 40 and 60 years of maturity. We make this choice to be consistent with the same division in time that we adopt with the mean surface temperature. The shifts will be introduced exactly at those three points in time. Computing the correlation using single instruments' time series we obtain what is reported in Table 7.

As regards the second passage, we rely on the estimation of the IPCC's scenarios for

Table 7: Historical correlations between air emissions and IRS with maturity 20, 40, 60 years in the period 01/01/2015 - 31/12/2021.

AIR EMISSIONS CORRELATIONS		
IRS 20Y	**IRS 40Y**	**IRS 60Y**
0.5247	0.5342	0.5330

future CO_2 emissions. From the dataset in [16], we recover the values and calculate the variations with respect to 2020 of the estimated emissions in 2040, 2060 and 2080. We lose here a bit of consistency: the periods are not exactly the same we consider for interest rates with maturity at 20, 40 and 60 years from now (for two years); the data includes only carbon dioxide, while our historical data entails different greenhouse gases; the emissions estimated from IPCC are global, instead our focus is only the European Union. Nevertheless, the major role in emissions is played by CO_2 and just a couple of years of difference does not have a substantial impact on the estimations. Hence, we recover the variations as in Table 8.

Table 8: CO_2 emissions variations from IPCC's AR6 in 2040, 2060 and 2080 with respect to the level of 2020.

CO_2 EMISSIONS VARIATIONS					
TIME	**SSP1-1.9**	**SSP1-2.6**	**SSP2-4.5**	**SSP3.7-0**	**SSP5-8.5**
2040	-0.74	-0.33	0.09	0.31	0.57
2060	-1.04	-0.74	-0.01	0.49	1.30
2080	-1.18	-1.08	-0.34	0.64	1.97

4.4.2 Temperature scenarios

Similarly, we proceed with the temperature scenarios. We compute the linear correlation between the daily average European temperature and the time series of IRS rate quotes at 20, 40 and 60 years. This is because IPCC defines each temperature scenario for three periods (2021-2040, 2041-2060, 2080-2100) and, hence, we select the corresponding instrument with the maturity at the end of the scenarios' period. Since there are no quotes for instruments with maturity up to 2100, for the last one we consider the beginning of the period. Notice that, for example, the IRS at 20 and 40 years mature in 2042 and 2062. However, we consider them in the previous scenario for practical reasons as there is no quoted IRS that matures 38 years from today. It is an adaptation of the available data within the assumptions of the scenarios. The results of the correlation are reported in Table 9.

Each scenario forecasts a different increase in temperature over a time period, based on

Table 9: Historical correlations between mean surface temperature and IRS with maturity 20, 40, 60 years in the period 01/01/2015 - 31/12/2021.

MEAN TEMPERATURE CORRELATIONS		
IRS 20Y	**IRS 40Y**	**IRS 60Y**
-0.0684	-0.0676	-0.0663

the underlying assumptions. To incorporate the variations in temperature into interest rate projections, we have to transform the increments into a percentage. To do so, we have to understand which is the starting point of the measurements, i.e. the global average temperature used as the basis for the scenarios. There is no direct reference in the IPCC report about the 1995-2014 global average temperature and neither for the pre-industrial period 1850-1900. However, from the ERA5 dataset, we know that the global average temperature for 1991-2020 is estimated at 14.4C, which is about 0.88C higher than the mean of 1850-1900. We can now recover the 1995-2014 basis as $13.52 + 0.82 = 14.34$C, with 0.82 being the difference between the two bases used in IPCC, and we use this average as a base to calculate the percentage increases in temperature since it is close to the period that we are studying.

As a remark, notice that we considered the daily average temperature in the EU, while these are global scenarios. This means that our results will be an approximation and biased in an optimistic way since the warming rate in Europe is currently higher. We can now use these percentage increments to evaluate the impact on interest rates.

Table 10: Mean temperature variations from IPCC's AR6 in 2021-2040, 2041-2060 and 2081-2100 with basis 1995-2014.

MEAN TEMPERATURE VARIATIONS					
TIME	**SSP1-1.9**	**SSP1-2.6**	**SSP2-4.5**	**SSP3.7-0**	**SSP5-8.5**
2021-2040	0.04	0.04	0.05	0.05	0.06
2041-2060	0.05	0.06	0.08	0.09	0.10
2081-2100	0.04	0.06	0.13	0.20	0.24

4.5 Benchmark curves construction

The distortion is introduced into the zero-rates so, to spread the effects, we have to compute again the zero-coupon bonds. We calculate them for each different scenario with respect to the distorted zero-rates with maturity at 20, 40 and 60 years. The last future value is, in our case, the zero-coupon bond corresponding to the zero-rate shifted at 60 years of maturity. The value was retrieved from the quote of IRS: since we use the quotes of IRS with maturity from 2 to 60 years, we recover all the corresponding values in a straightforward way. Going further, we compute the inverse of the 6x12 and 12x18 FRAs, while in the bootstrapping we used 7 different FRA quotes. This is because they are not directly linked to the distortion introduced in the IRS quotes, but we recover their values by interpolation. The last instrument that we used in bootstrapping was a Deposit. In this case, we cannot invert the formula to obtain the previous value. However, we have that the Depo rate changes accordingly to satisfy equation (2). Once we retrieved all the values, we optimize the problem as presented before. After that, we recompute the new zero-rates and retrieve the benchmark curves by mean of Monotonic Cubic Spline interpolation. All the procedure is performed for each scenario.
The variables that we study in this research, surface mean temperature and emissions, are strictly connected. We know that high GHG emission concentrations directly influence the global temperature, but the inverse relationship does not hold. In fact, IPCC's scenarios are primarily based on future emissions, while temperature scenarios are a consequence of the total amount of radiative forcing at the end of 2100. Given this strict link, we are interested in evaluating the joint effects of temperature and emissions on interest rates. To be more precise, we have to consider the case of temperature variations' impact once the influence of GHG emissions has taken place, otherwise, we would consider the effects of the variables as if they were independent of each other. Therefore, we focus on the outcomes from a variation in temperature conditioned to changes in emissions. Put into practice, we start from the distortion of the market zero rates with maturity at 20, 40 and 60 years relative to the emission scenarios as in equation (1). We then apply the reverse bootstrapping and recover the five benchmark curves, obtaining the impacts of emissions on our term structure. Going further, to account also for the temperature, we shift again the zero rates for each equivalent temperature scenario. We take now those from the corresponding benchmark emissions curve and not from the original one. For example, if we consider scenario SSP1-1.9 at 20 years of maturity, we would introduce the distortion in the zero rates as

$$\Delta^{em}R(t,20) = R(t,20) + R(t,20) \times 0.5247 \times (-0.74), \tag{13}$$

where we use the values from Table 7 and 8. The same must be done for the zero rates at 40 and 60 years of maturity. After recovering the entire SSP1-1.9 benchmark curve for emissions, we apply the same procedure considering the relative temperature scenario SSP1-1.9 at 20 years of maturity. Therefore

$$[\Delta^{temp}R(t,20)|em] = \Delta^{em}R(t,20) + \Delta^{em}R(t,20) \times (-0.0684) \times 0.04 \tag{14}$$

where we use the values from Table 9 and 10. We repeat the same passages at 40 and 60 years of maturity. Through reverse bootstrapping, we recover the final benchmark curve representing the temperature scenarios conditioned to the emission ones. This is done for each SSP's scenario.

In Figure 2, we plot the five conditioned benchmarks against the original yield curve derived from the market. As we can see, the benchmark curves lie above or below the current zero curves depending on the assumptions, and consequent variations, under the different scenarios. Those that consider no or limited climate intervention (SSP5-8.5 and SSP3-7.0) predict an increase in the rates in the long term, while those that assume heavy limitation of emissions (SSP1-1.9 and SSP1-2.6) result in a decrease in the term structure. Scenario SSP2-4.5 curve is the closest to the original one due to the similar climate change assumptions. In fact, it represents a situation where no other modification is introduced with respect to what is employed currently, at least until the middle of the century. The benchmark curves are coherent with the fact that, in case of higher uncertainty, the market reflects the risk by increasing the rates. On the contrary, if policy interventions succeed in achieving huge reductions in GHG emissions, the market is more stable and the rates are lower. Hence, our distortions display a sort of riskiness implied by that particular scenario.

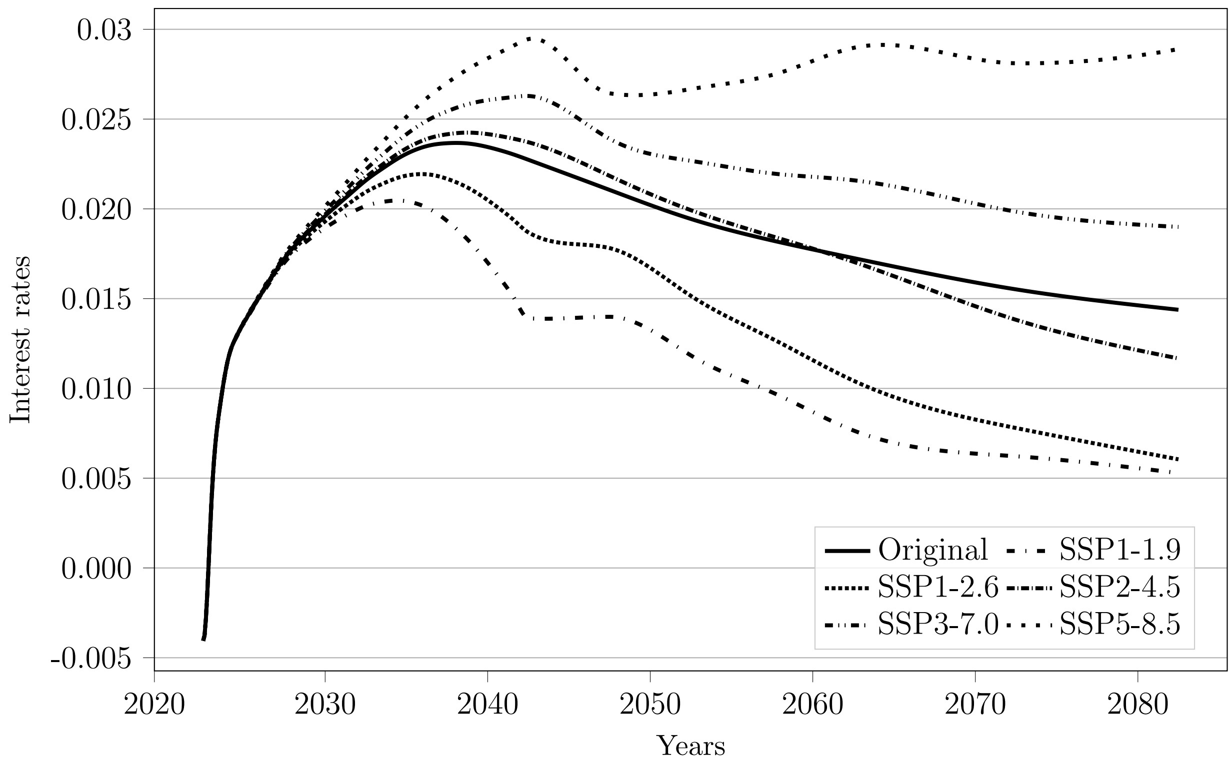

Figure 2: Temperature benchmark curves conditioned to emissions scenarios for each SSP scenario, plotted against the original market zero curve.

4.6 The climate-Vasicek model implementation

Finally, we can set up the climate-Vasicek model that we formalized. Our aim is to recover the values of the parameters that replicate the shape of each benchmark curve.

In practice, the calibration of our model starts with the simulation of the future values for the rates y and for the climate change variable c. We perform $n = 10^3$ different simulations for each factor. Despite the number may look smaller, it is high enough to achieve encouraging results and finalize the optimization in a reasonable amount of time. The next step is to split again the benchmark curves into three parts, at the same points as we did during their construction. In fact, each part is dominated by the shift we introduced at one point, using that specific correlation. To be consistent as much as possible with the idea under the benchmark curves, we use three different coefficients of correlation also when performing the optimization of the model. In this case, however, we use the correlation considering the average of the interest rates instruments for each period, hence for the first 20 years, between 21 and 40 and 41 to 60. Remember that we built benchmark curves that account for the variations in temperature implied by the CO_2 emissions, where we performed a double distortion to take care of this conditioning. In the climate-Vasicek model, we cannot proceed in the same way. Instead, we need to find a measure of dependence among the three variables at the same time. Since linear correlation can be calculated only between two variables, we rely on copula functions. From Sklar's theorem, we know that any multivariate joint distribution may be rewritten as a univariate marginal distribution function and a copula. Indeed, we use this relation to study the inter-correlation structure between the random variables. Among the different parametric copula families existing, we select the Frank copula. It belongs to the family of Archimedean copulas, which admit an explicit formula and make use of just one parameter to control the strength of dependence. This choice is led by the feature of this particular copula of modelling the dependence of middle values, contrary to other Archimedean copulas. In fact, we can measure in this way the symmetric dependence between variables rather focusing on the dependence in the tails. Without going into other details, we refer to [19] for further information. We recover the α parameter defining the Frank copula on our three-time series using R. It can be then transformed into a measure of ordinal association, i.e. the Kendall's τ coefficient, through the following relationship:

$$\tau = 1 + \frac{4}{\alpha}(D_1(\alpha) - 1), \quad \text{with} \quad D_1(\alpha) = \frac{1}{\alpha}\int_0^{\alpha} \frac{t}{e^t - 1}dt.$$

Once we recover Kendall's τ, we can convert it into Pearson's ρ to finally obtain a synthetic value for the dependence among interest rates, temperature and emissions to use inside our affine model. We report the coefficients in Table 11. Compared to the ones for temperature and emissions, we notice that they are reasonable: they are lower than those with emissions probably because of the negative dependence effect of temperature and interest rates and temperature and emissions. As regards the benchmark curves of only the temperature or emissions, we simply use the correlation with the average interest rates quoted in each time period considered (see Section 4.2).

Table 11: Pearson correlation coefficients between interest rates, emissions and mean temperature for the period 2015-2021 recovered from the Frank copula's Kendall's τ.

TEMPERATURE AND EMISSIONS		
6M - 20Y	**20Y - 40Y**	**40Y - 60Y**
0.4039	0.3899	0.3909

Our calibration is thus made on one part at a time, not for the entire curve at once. Although this choice is made primarily for coherence, it is also a common procedure in the calibration of the Vasicek model. It is well known that its simplicity, due to the constant parameters, could lead to poor results. Indeed, to improve the fitting, the curve can be split and calibrated for each part: the so-called calibration by compartments. This happens in particular when the shape of the curve assumes particular forms, as in our case. If we consider the extreme scenarios, SSP1-1.9 and SSP5-8.5, we can see that there are various humps and dips impossible to model with the same constant parameters for the entire curve. After some trials, we find out that the best splitting is at every 10 years. This is in particular true for the first section of the curve, which otherwise is underestimated by the calibration due to the high hump. We thus calibrate the model for 10 years at a time, using $\tau = 2$ years to avoid excessive over-fitting.

4.7 Results discussion

Finally, we can analyse and discuss the results of the calibration performed as described above. The benchmark curve of scenario SSP1-1.9 represents the Paris Agreement goal, with strong mitigation actions and a temperature increase limited to 1.5C. Therefore, it would be the best case from a climatic point of view. In fact, the curve is noticeably lower than the current market curve. We can see that the effects of the climate variables start to be more visible from the tenth year onwards. In 2042, the point at which we introduced the first shift, the benchmark curve is about 0.8% lower than the current one and until 2080 it slowly decreases to 0.5%, hence almost 1% lower than the value on the market curve. The SSP1-1.9 curve is not perfectly smooth, in 2042 there is almost an angle, which usually is not a desirable feature in yield curves. This point is probably due to the optimization made during the reverse bootstrapping. The assumption of the fixed forward rates did not leave enough space for zero-coupon bonds to accommodate nicely. As regards the calibration, the results plotted over the original benchmark curve in Figure 3 show that in the long term the model is able to capture almost perfectly the zero rates. The same is not true for the first 20 years of the term structure, where the points estimated by the model are sometimes not so close to the original one. In particular, at 10 and 12 years we notice that the model underestimates and overestimates respectively the zero rates. Consider, however, that these two points represent the calibration performed on two different sections of the curve.

Another interesting point is again the one at 20 years. Here, the model overestimates the point and makes the curve smoother than the original one. Of course, the Vasicek model, even with two factors, cannot reproduce such a shape. In Table ?? we report the estimated parameters for each section of the curve (every 10 years). It is harder to interpret these values than in the one-factor case, however, we can provide some remarks. The values for κ_i are correctly always positive, with κ_1 being very high for the first section with respect to the other values of κ_i. From the one-factor model, we know that this parameter usually represents the speed of reversion and it should be interpreted together with the value of σ_i^2. Their ratio represents the long-term variance for each factor. We can notice that the high values for σ_i^2 are balanced by the coefficients of κ_i. The values for the mean reversion parameters θ_i are generally positive for one factor and negative for the other, indicating a sort of compensation between them. The market price of risk is positive for both factors.

The SSP1-2.6, SSP2-4.5 and SSP3-7.0 scenarios are smoother and look like real zero-rate curves. SSP1-2.6 scenario represents the target of a maximum increase in temperature of 2C, thus requiring a substantial cut in GHG emissions by the second half of this century. On average, it is lower than the current curve by about 0.5% and it increases in time up to almost 1% in 2080. This scenario is very similar to the SSP1-1.9 one, differing only by the timing of the effects but reaching almost the same results over a long period.

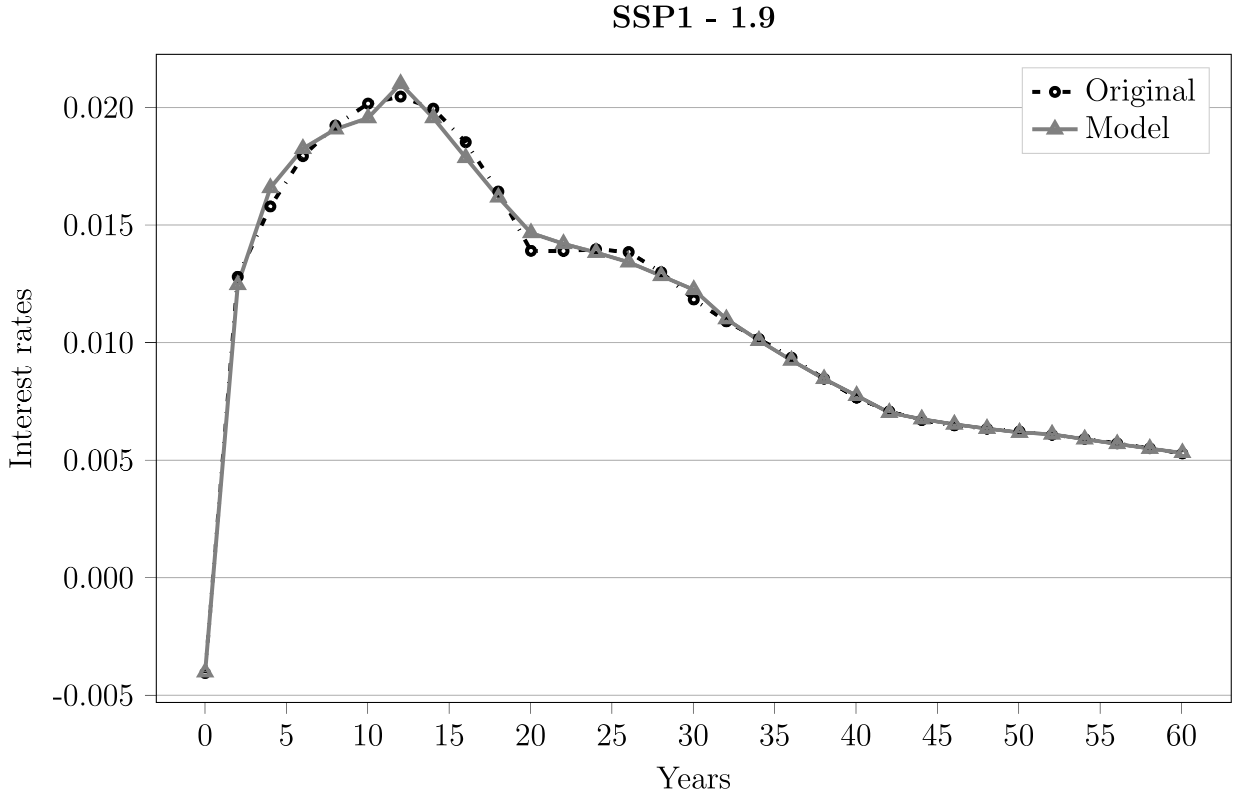

Figure 3: Temperature SSP1-1.9 benchmark curve conditioned to emissions scenario and correspondent model curve.

Table 12: Climate-Vasicek model's parameters for the SSP1-1.9 scenario for temperature given emissions at each 10 years.

SSP1 - 1.9							
κ_1	κ_2	θ_1	θ_2	λ_1	λ_2	σ_1^2	σ_2^2
41.72	15.81	-4.85	4.89	1.49	1.57	0.06	5.86
22.76	18.22	4.55	-2.79	0.47	2.73	3.14	15.34
18.80	17.92	3.45	-1.58	0.70	2.45	2.00	11.35
7.54	16.77	-3.21	6.80	2.96	0.03	29.05	8.17
17.65	14.27	2.32	-0.43	0.28	2.66	2.71	14.94
14.43	13.07	1.93	0.52	1.01	2.75	5.73	16.79

On the contrary, the SSP3-7.0 scenario is the first that does not consider an immediate reduction in GHG emissions and thus entails a higher increase in temperature over time. Accordingly, it always lies above the market curve for about 0.5% over the entire period in the examination. A middle-of-the-road situation is clearly visible from scenario SSP2-4.5. The initial part of the curve is above the current one notwithstanding that the emissions are expected to maintain the same levels as today. From 2060, the curve diminishes corresponding to the reduction of GHG emission, however, never reaches the net zero. This scenario is very similar to the current curve and it lies below or above it for less than 0.4%. In the long term, the model predicts the values very close to the real ones from the

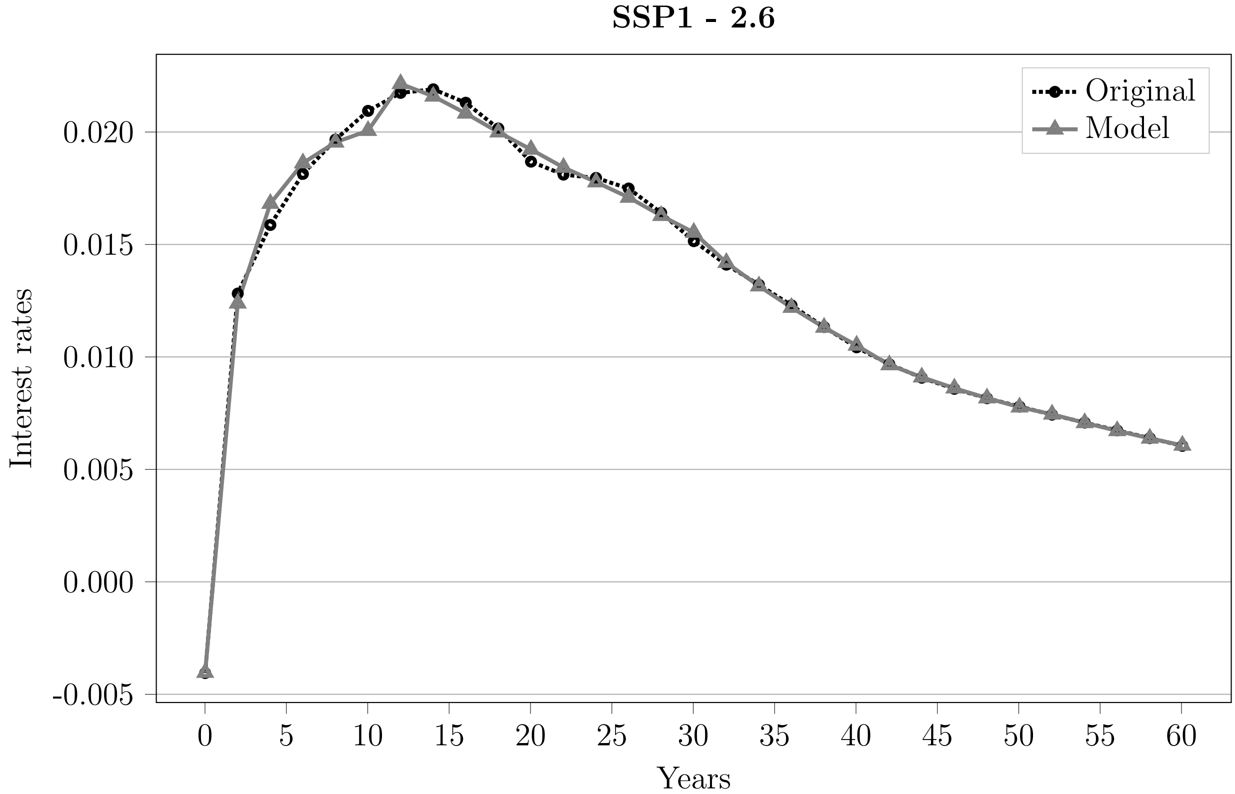

Figure 4: Temperature SSP1-2.6 benchmark curve conditioned to emissions scenario and correspondent model curve.

benchmark curves. Only the SSP1-2.6 curve from the model looks slightly smoother than the original. Despite this, all three calibrations result in an over- and underestimation of the points at 10 and 12 years. Again, this is because it is the point at which we separated the calibration. In Tables 13, 14 and 15 we report the1 values of the calibrated parameters. The values of θ_i do not follow a particular pattern, they differ in sign and size for all three scenarios. We should consider them in connection with the market price of risk κ_i and σ_i^2 to compute the actual mean reversion factor under the risk-neutral measure. The values of σ_i^2 are usually higher in the second half of the curve, whereas in the SSP2-4.5 scenario reach the value of 61.59. These values indicate a substantial level of randomness in the system. By looking at the κ_i, we notice that their high values are capable of stabilizing the fluctuations in the long term (remember the long-term variance computed as $\sigma_i^2/2\kappa_i$).

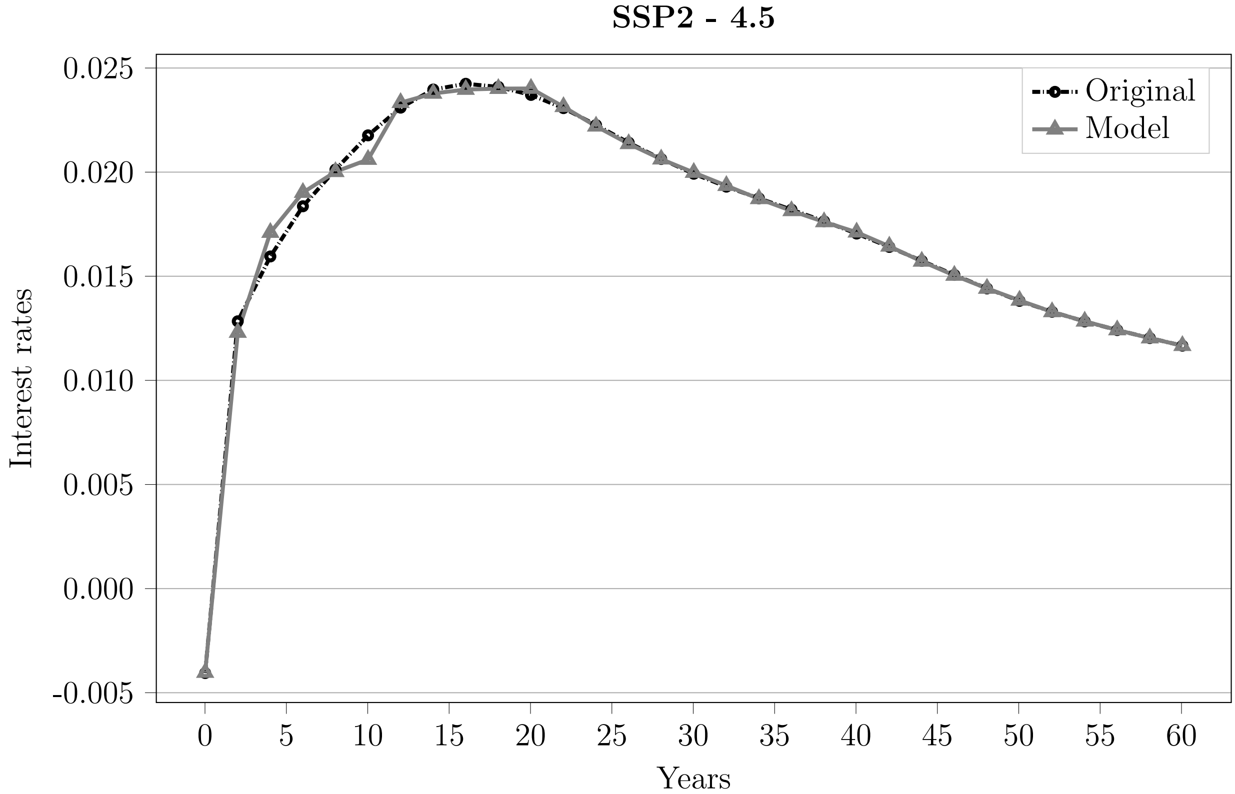

Figure 5: Temperature SSP2-4.5 benchmark curve conditioned to emissions scenario and correspondent model curve.

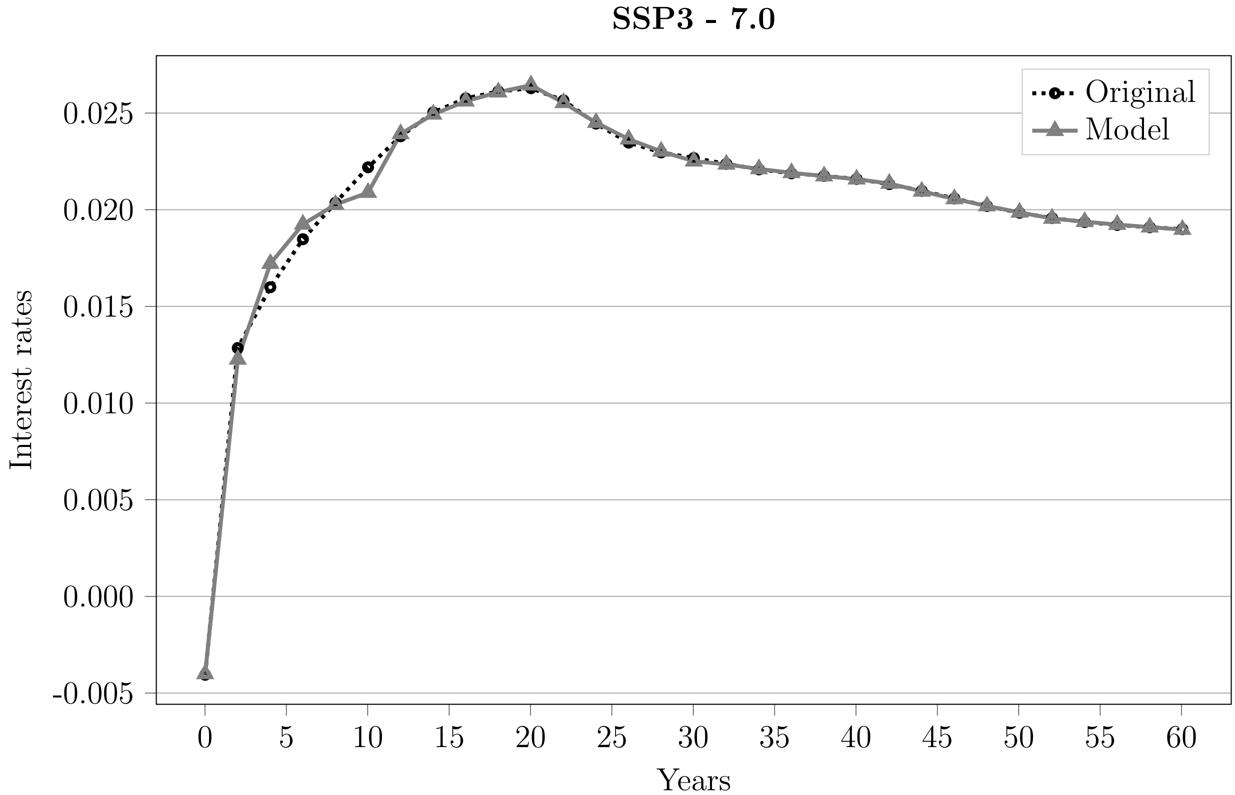

Figure 6: Temperature SSP3-7.0 benchmark curve conditioned to emissions scenario and correspondent model curve.

Table 13: Two-factor Vasicek model's parameters for the SSP1-2.6 scenario for temperature given emissions at each 10 years.

SSP1 - 2.6							
κ_1	κ_2	θ_1	θ_2	λ_1	λ_2	σ_1^2	σ_2^2
44.57	16.34	-4.78	4.89	1.50	1.59	1.25	5.42
27.92	22.42	3.15	-1.73	0.75	2.20	1.95	9.18
17.37	15.46	3.57	-1.43	0.59	2.60	2.95	13.79
7.77	18.12	-3.96	8.55	4.06	-0.45	46.39	9.54
8.24	15.45	-4.26	6.87	2.34	0.29	22.76	6.38
7.65	14.64	-3.47	7.46	3.75	0.09	38.98	6.42

Table 14: Climate-Vasicek model's parameters for the SSP2-4.5 scenario for temperature given emissions at each 10 years.

SSP2 - 4.5							
κ_1	κ_2	θ_1	θ_2	λ_1	λ_2	σ_1^2	σ_2^2
43.60	15.31	-5.28	5.28	1.54	1.63	3.14	6.97
33.36	32.15	0.21	0.71	0.91	1.86	0.59	4.94
19.19	17.17	2.59	-0.61	0.69	2.96	2.55	19.32
13.08	15.62	-0.03	2.27	1.32	2.13	6.49	11.67
7.81	17.02	-3.87	9.64	5.04	-0.61	61.59	8.85
8.99	18.56	-6.08	10.69	5.20	0.43	45.72	1.80

Table 15: Climate-Vasicek model's parameters for the SSP3-7.0 scenario for temperature given emissions at each 10 years.

SSP3 - 7.0							
κ_1	κ_2	θ_1	θ_2	λ_1	λ_2	σ_1^2	σ_2^2
45.55	15.11	-4.92	5.17	1.56	1.50	8.53	1.59
31.27	27.58	0.78	-0.13	0.81	2.04	0.06	4.77
17.10	17.14	3.39	-1.04	0.61	2.85	3.48	17.35
22.40	18.73	2.47	-1.32	0.73	2.94	0.91	14.49
11.39	12.68	0.63	2.66	1.77	2.06	11.60	15.96
17.25	14.35	1.66	0.003	0.53	2.84	1.71	15.80

The worst-case scenario SSP5-8.5 predicts no climate policy and intervention on GHG emissions. Thus, it is expected that they will consistently grow over time and, accordingly with them, the mean temperature too. The benchmark curve displays a significant increase in time of the zero-rates, reaching almost 1.5% above in 2080. This represents the substantial rise in uncertainty in the market due to the worsening of climate conditions. Similarly to what happened with the SSP1-1.9 scenario, also here the curve is not particularly smooth. The points at which we introduced the shifts can be recognized: there are three visible humps in 2042, 2062 and 2082. Regarding the calibration, as usual, the long-term part is almost perfectly reproduced by the model. From today up to 40 years from now, instead, the zero rates are alternatively over- or underestimated by the model. Here, the point at 10 years is likely the same for the model and for the benchmark curve, while the 12th-year point underestimates the original point. Another section less well fitted is that between 20 and 30 years, where again we have some small inconsistencies. In Table 16 we synthesize the estimated parameters for this scenario. The coefficients for κ_i are on average equal to 15 for all the parts of the curve, excluding the first twenty years where they are consistently higher. The θ_1 is always positive, with an exception for the

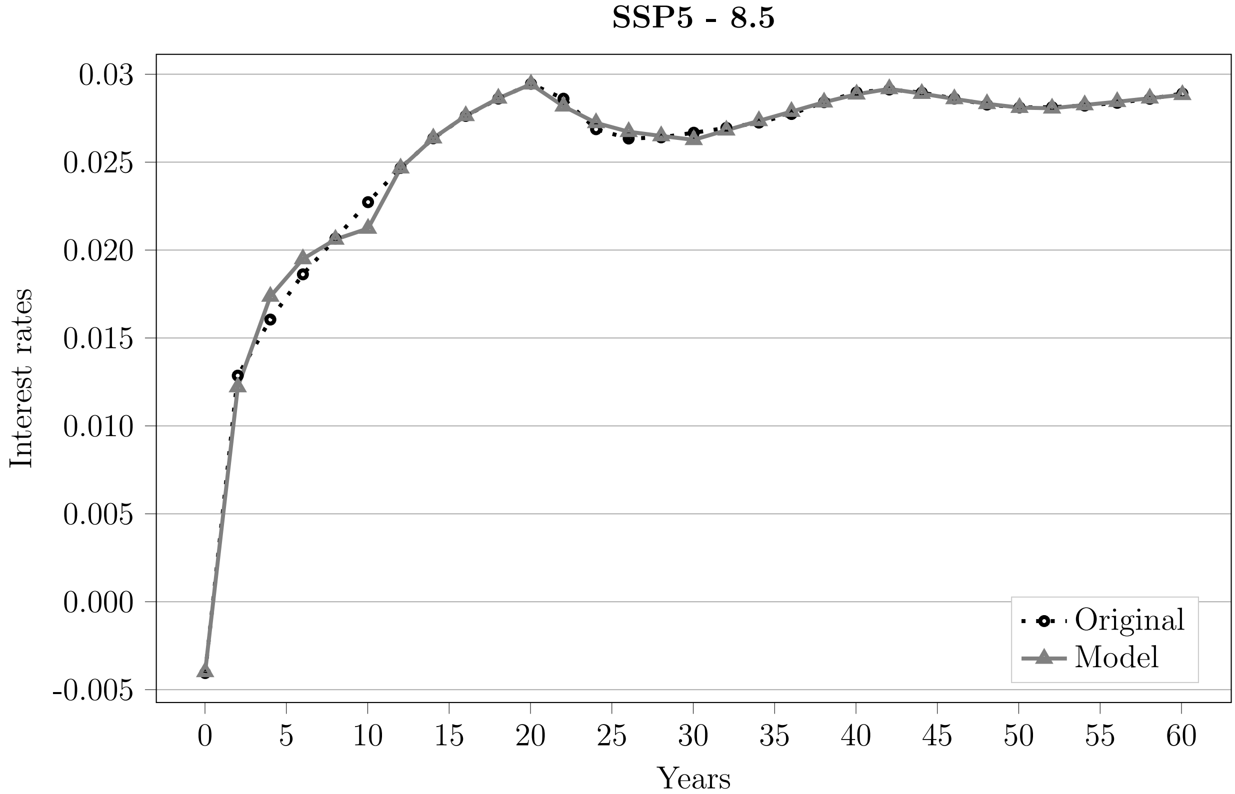

Figure 7: Temperature SSP5-8.5 benchmark curve conditioned to emissions scenario and correspondent model curve.

Table 16: Climate-Vasicek model's parameters for the SSP5-8.5 scenario for temperature given emissions at each 10 years.

SSP5 - 8.5							
κ_1	κ_2	θ_1	θ_2	λ_1	λ_2	σ_1^2	σ_2^2
44.64	15.10	-5.19	5.30	1.56	1.63	2.34	4.72
24.46	32.65	7.94	-7.43	1.09	4.07	1.24	14.25
18.15	16.97	3.04	-1.17	0.77	2.29	2.17	9.76
13.81	15.38	1.61	-0.59	1.21	2.21	0.19	3.69
14.30	13.47	2.14	0.41	0.92	2.79	4.40	16.30
13.18	15.04	1.54	-0.68	0.94	2.17	0.02	3.57

first 10 years, and respectively θ_2 compensates with an opposite sign almost every time. The λ_i are positive, with λ_2 always greater than λ_1 and the same relation holds for σ_i^2. There is a greater fluctuation in the randomness in sections 10-20 years and 40-50 years.

In conclusion, the calibrations for each scenario produced in general good fittings of the curves. A higher number of simulations however should produce even better results. We found high values for the parameters estimated. This is probably due to the reduced number of points calibrated, the short time periods considered for each section and, above all, the randomness in the underlying factors.

4.8 The BCC approach as a policy tool

Over the years, various policies are pursued at a national or international lever. It is clear that the long-term nature of climate change requires interventions that can anticipate the effects and prepare the economy for the future. The environmental and climate change concerns are however a significant source of instability and a multiplier of threats. The transition to a low-carbon economy will restructure society as it is known today. For these reasons, the EU already started considering the implications of climate policies when making decisions and taking action. In the short term, under the 2030 climate and energy framework, the EU aims to reduce the GHG emissions of at least 40% to the levels of 1990, account for at least 32% share of renewables, enhance the energy efficiency of at least 32.5%. Notice that the EU's NDC of 2020, however, slightly modified these aims and targets now an emission reduction of at least 55% by 2030 [9]. In line with the commitment of the EU to the Paris Agreement, the EU set in 2020 the European Green Deal (EGD), which corresponds to various policy initiatives directed to tackle climate change with the objective of shifting the current economy to its enhanced version, increasing the fairness, wealth and competitiveness. The core purposes are achieving zero net GHG emissions by 2050 and decoupling the economic expansion from the use of resources. Furthermore, the EGD attempts to privilege natural capital, the well-being of people more subject to environmental risks, while always worrying about realizing a transition as equal and inclusive as possible. It acknowledges the variety of subjects, sectors and areas affected, thus providing more attention to those who must face up the hardest challenges [7].

Despite attempts at mitigation, climate change continues to put a substantial burden on Europe and thus the Commission has to reinforce its approach for adaptation. It will be crucial to ensure that investors, insurers, enterprises, towns, and individuals can access data and create tools to incorporate climate change into their risk management techniques throughout the EU. The risks related to the climate and the environment will be controlled and assimilated into the financial system. This entails assessing the adequacy of the current capital requirements for green assets and better integrating such risks into the EU prudential framework. Via the financial system, it would be possible to improve resiliency to climate and environmental threats, particularly in terms of the dangers and harm caused by physical natural disasters [7].

Given these remarks, the BCC approach can be seen as an helpful tool to assess the different possible climate effects on interest rates. Given the relevance of the fixed-income market, it would be essential to size the impact of an increase in climate risk and be able to formulate different scenarios. Although the yield curve is subject to a continuous repricing in the market, in the long term it matters more the overall direction than the daily volatility. Since the movements that we introduce with the BCC approach are exclusively dependent on the size of climate risk, ceteris paribus, we can efficiently separate it from other factors that normally influence interest rate. Therefore, monitoring the course of interest rates could potentially improve the understanding of long-term expectations. Consequently, this approach can be used to guide the market towards the right direction: understanding the potential impacts is the first step towards actions that can limit the increase in climate risk, as the EU aims.

5 Conclusions

This paper attempted to model the impacts of climate change on the term structure of interest rates by using the proposed BCC approach. Our innovative method leads to a coherent future distortion in rates, linked to the forecast of climate risk, that can be modelled by introducing a climate change factor inside a short rate model. In particular, the construction of the so-called benchmark curves, representing each one a different climate scenario, allowed us to calibrate on them the parameters of a Climate Vasicek model. Applied to the EUR market, our method demonstrates that the term structure can be influenced positively or negatively by the effects of climate change. The rates might increase more than 1% due to the uncertainty caused by the introduction of climate risk under the worst case scenario. Instead, in case of policy interventions that substantially limit GHG emissions, the term structure decreases, displaying the implications of lower climate risks perceived in the future. The size of the variations is greater in the long term due to the forward-looking nature of climate. This evidence shows how the BCC approach could help the EU to favour the implementation of more sustainable measures to deal with climate change.

The general methodology that we proposed can be adapted and investigated further. First of all, there are different variables that can be substituted e.g. the climate scenarios, the climate change variables, the short-rate model. Our selection of factors was driven by the focus on Eurozone, however it is possible to easily change them accordingly to the chosen market. Second, the potential shift in the sentiment of investors and consumers due to the Covid-19 pandemic should be deepened. It is worth to look into it as it could represent the turning point in how the market prices climate risk and would be essential for policy makers to monitor. Finally, it would be interesting to model the dynamics of the climate change variables to account for their specific characteristics also in the model, for example considering the seasonal effect that those kind of variables usually have. However, this will add more complexity.

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A Appendix: Bootstrapping instruments

Table 17: Bootstrapping instruments selected for the discounting OIS curve at 8th July 2022.

BOOTSTRAPPING INSTRUMENT	TENOR	RATE (%)	START DATE	END DATE
DEPO ON	1D	-0.4000	July 8th, 2022	July 11th, 2022
DEPO TN	1D	-0.4000	July 11th, 2022	July 12th, 2022
DEPO SN	2D	-0.4000	July 12th, 2022	July 13th, 2022
OIS SW	1W	-0.5800	July 12th, 2022	July 19th, 2022
OIS 2W	2W	-0.5800	July 12th, 2022	July 26th, 2022
OIS ECB JUL	Fix	-0.2890	July 28th, 2022	Sept 15th, 2022
OIS ECB SEPT	Fix	0.2290	Sept 15th, 2022	Nov 3rd, 2022
OIS ECB NOV	Fix	0.6250	Nov 3rd, 2022	Dec 22nd, 2022
OIS ECB DEC	Fix	0.9000	Dec 22nd, 2022	Feb 9th, 2023
OIS 15M	15M	0.8220	July 12th, 2022	Oct 12th, 2023
OIS 18M	18M	0.9280	July 12th, 2022	Jan 12th, 2024
OIS 21M	21M	1.0050	July 12th, 2022	April 12th, 2024
OIS 2Y	2Y	1.0620	July 12th, 2022	July 12th, 2024
OIS 3Y	3Y	1.2060	July 12th, 2022	July 14th, 2025
OIS 4Y	4Y	1.3420	July 12th, 2022	July 13th, 2026
OIS 5Y	5Y	1.4710	July 12th, 2022	July 12th, 2027
OIS 6Y	6Y	1.5680	July 12th, 2022	July 12th, 2028
OIS 7Y	7Y	1.6510	July 12th, 2022	July 12th, 2029
OIS 8Y	8Y	1.7290	July 12th, 2022	July 12th, 2030
OIS 9Y	9Y	1.8040	July 12th, 2022	July 14th, 2031
OIS 10Y	10Y	1.8770	July 12th, 2022	July 12th, 2032
OIS 11Y	11Y	1.9430	July 12th, 2022	July 12th, 2033
OIS 12Y	12Y	2.0060	July 12th, 2022	July 12th, 2034
OIS 13Y	13Y	2.0560	July 12th, 2022	July 12th, 2035
OIS 14Y	14Y	2.0920	July 12th, 2022	July 14th, 2036
OIS 15Y	15Y	2.1120	July 12th, 2022	July 13th, 2037
OIS 16Y	16Y	2.1250	July 12th, 2022	July 12th, 2038
OIS 17Y	17Y	2.1290	July 12th, 2022	July 12th, 2039
OIS 18Y	18Y	2.1240	July 12th, 2022	July 12th, 2040
OIS 19Y	19Y	2.1150	July 12th, 2022	July 12th, 2041
OIS 20Y	20Y	2.1020	July 12th, 2022	July 14th, 2042
OIS 25Y	25Y	2.0200	July 12th, 2022	July 12th, 2047
OIS 30Y	30Y	1.9330	July 12th, 2022	July 12th, 2052
OIS 40Y	40Y	1.8250	July 12th, 2022	July 12th, 2062
OIS 50Y	50Y	1.7350	July 12th, 2022	July 12th, 2072
OIS 60Y	60Y	1.6530	July 12th, 2022	July 13th, 2082

Table 18: Bootstrapping instruments selected for the 6 month curve at 12th July 2022.

BOOTSTRAPPING INSTRUMENT	**RATE(%)**	**START DATE**	**END DATE**
DEPO 6M	0.39	July 12th, 2022	Jan 12th, 2023
FRA 1x7	0.59	Aug 12th, 2022	Feb 13th, 2023
FRA 2x8	0.828	Sept 12th, 2022	March 13th, 2023
FRA 3x9	1.004	Oct 12th, 2022	April 12th, 2023
FRA 4x10	1.165	Nov 14th, 2022	May 15th, 2023
FRA 5x11	1.3	Dec 12th, 2022	June 12th, 2023
FRA 6x12	1.398	Jan 12th, 2023	July 12th, 2023
FRA 12x18	1.682	July 12th, 2023	Jan 12th, 2024
IRS6M 2Yrs	1.3	July 12th, 2022	July 12th, 2024
IRS6M 3Yrs	1.461	July 12th, 2022	July 14th, 2025
IRS6M 4Yrs	1.604	July 12th, 2022	July 13th, 2026
IRS6M 5Yrs	1.735	July 12th, 2022	July 12th, 2027
IRS6M 6Yrs	1.839	July 12th, 2022	July 12th, 2028
IRS6M 7Yrs	1.925	July 12th, 2022	July 12th, 2029
IRS6M 8Yrs	2.006	July 12th, 2022	July 12th, 2030
IRS6M 9Yrs	2.083	July 12th, 2022	July 14th, 2031
IRS6M 10Yrs	2.156	July 12th, 2022	July 12th, 2032
IRS6M 11Yrs	2.217	July 12th, 2022	July 12th, 2033
IRS6M 12Yrs	2.272	July 12th, 2022	July 12th, 2034
IRS6M 13Yrs	2.314	July 12th, 2022	July 12th, 2035
IRS6M 14Yrs	2.342	July 12th, 2022	July 14th, 2036
IRS6M 15Yrs	2.353	July 12th, 2022	July 13th, 2037
IRS6M 16Yrs	2.355	July 12th, 2022	July 12th, 2038
IRS6M 17Yrs	2.347	July 12th, 2022	July 12th, 2039
IRS6M 18Yrs	2.331	July 12th, 2022	July 12th, 2040
IRS6M 19Yrs	2.31	July 12th, 2022	July 12th, 2041
IRS6M 20Yrs	2.285	July 12th, 2022	July 14th, 2042
IRS6M 25Yrs	2.151	July 12th, 2022	July 12th, 2047
IRS6M 30Yrs	2.021	July 12th, 2022	July 12th, 2052
IRS6M 35Yrs	1.923	July 12th, 2022	July 12th, 2057
IRS6M 40Yrs	1.843	July 12th, 2022	July 12th, 2062
IRS6M 50Yrs	1.703	July 12th, 2022	July 12th, 2072
IRS6M 60Yrs	1.609	July 12th, 2022	July 13th, 2082