Optimal Dynamic Contracts and Pollution

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Abstract

We examine optimal dynamic contracts when the firm's production generates harmful pollution undermining its productivity. The optimal contract rewards for financial performance and penalizes pollution. The combination of both contract sensitivities incentivizes the agent's effort and environmental (pollution abating) investment. When the accumulated pollution exceeds a threshold, the contract sensitivity to financial performance drops, its sensitivity to pollution emerges, and environmental investment increases with pollution. In an economy with a continuum of polluting firms, contracting on firm pollution improves the welfare of the principal and the agent. Calibrating the model to the U.S. economy, we show that the aggregated pollution is reduced by 38.4% if all firms contract on their own pollution.

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Over the past few decades, pollution has become an increasingly important matter of concern for individuals, corporations, regulators and politicians. Evidence shows pollution has a significant effect on the productive capabilities of firms and the Gross National Product (GNP) of nations. Evidence also shows that firms implementing policies designed to improve sustainability derive reputation benefits, that may lead to valuation gains taking the form of Green premia. This paper examines the relevance of pollution for managerial contracts. It shows the optimal contract rewards for environmental performance, examines optimal managerial decisions and studies aggregate implications.

Pollution takes many forms, with air, soil and water pollution being the most common ones and air pollution the most concerning one due to its health effects. Air pollution, comprising indoor and outdoor pollution, is estimated to have contributed to 11.65% of global death in 2019.[1] Outdoor air pollution, mainly resulting from emissions of Greenhouse Gases (GHG), particulate matter (PM) and ozone concentrations, is the leading factor behind 7.8% of global death.[2] Air pollution has increased steadily over the last century. Concentrations of GHG and other forcing agents such as aerosols, reached 460 parts per million (ppm) of CO2 equivalents (CO2e) in 2019, rising from 287.6 in 1900.[3] Air pollution varies significantly across cities and countries. Concentrations of PM2.5, for instance, stood at 90.87 micrograms per cubic meter in India vs 5.86 in Finland, in 2017.[4] Economic damages from air pollution have grown in magnitude over the years. For 2019, the induced global cost is estimated at $8.1 trillion, or 6.1% of the global GDP.[5] Concerns about pollution have similarly grown over time, leading to calls for mitigating actions and eventual agreements on measures at the bi-annual Conferences on Climate Change (COP) sponsored by the United Nations. Some companies, e.g., Xcel, have responded to these concerns by including climate change and emission metrics in executive compensation.[6]

In this paper, we examine the impact of pollution and concerns about pollution on decision-making within a firm and their implications for the environment beyond the firm. Our model relies on multiple ingredients, pertaining to the operations of the firm and the

[1] Air pollution is a risk factor for various health conditions including heart disease, stroke, respiratory infections, lung cancer and chronic obstructive pulmonary disease (COPD).

[2] Outdoor air pollution in Our World in Data.

[3] European Environment Agency.

[4] Particulate matter exposure in 1990 vs. 2007 from Our World in Data.

[5] World Bank. 2022. The Global Health Cost of PM2.5 Air Pollution : A Case for Action Beyond 2021. International Development in Focus; Washington, DC: World Bank.

[6] News report by StarTribune.

preferences of agents. There are three ingredients at the level of operations. First, the productive activities of the firm generate emissions, which along with background pollution, impede productivity once some threshold is reached. Second, the manager of the firm, in addition to deciding on an effort level, can also engage in mitigating actions, such as investing in emission-reducing technologies (carbon capture), improving maintenance of existing machines and equipment, setting up monitoring devices and controls, and cleaning up spills and chemical contamination on firm property. Such actions provide control over the emissions of the firm and the stock of pollution within the confines of the firm. Third, the endogenous pollution of any single firm blends together with the pollution generated by other firms, leading to an aggregate level of environmental pollution. The resulting aggregate pollution becomes the background pollution and feeds back into the productivity of each firm and affects everyone's decision-making. There are two ingredients pertaining to the agents involved. The risk averse manager of the firm, the agent, who operates on firm grounds, bears health consequences for the pollution generated. He may also have concerns for the safety and well-being of the workers and employees of the firm under his management, and/or the well-being of local residents. Likewise, the risk neutral owner of the firm, the principal, has self-health concerns and/or concerns for the well-being of employees or others exposed to the pollution generated. She designs a contract to hire the agent and optimize her net benefits from ownership.

Our main results are fivefold. First, we show the optimal contract rewards not only for financial performance, but also for environmental performance. The contract loads positively on the firm cash flow and negatively on the pollution generated by the firm. It does not load on background pollution resulting from the actions of other firms. Second, we show that the agent's effort is affected by the level of pollution. Once pollution reaches a threshold, the effort jumps down, then continues to decline (continuously) as the level of pollution increases. The contract sensitivity to cash flow, similarly, experiences a downward jump at the threshold then continues to decline (continuously). Third, we show that the agent invests in pollution-mitigating activities once the pollution level hits the above-mentioned threshold. At that threshold, the contract becomes sensitive to (penalizes for) environmental pollution, providing the agent with incentives to invest in pollution abatement. Investment then increases (continuously) as the level of pollution keeps increasing. At the same time, however, the magnitude of the contract sensitivity to pollution decreases. This behavior follows from the fact that investment is driven by the ratio of contractual exposures to environmental and financial performances and that the exposure to financial performance decreases as pollution increases, thus leading to an increase in the relative performance exposure. Fourth, we relate the pollution generated by a single firm, operating in an economy with a continuum of identical firms, to the aggregate ambient pollution. An equilibrium exists if each firm behaves optimally given a conjectured aggregate pollution and the resulting aggregate pollution matches the conjecture. An equilibrium is stationary if the distribution of aggregate pollution is time-invariant. We obtain a stationary equilibrium by solving numerically a coupled system composed of a Hamilton-Jacobi Bellman (HJB) equation and a Fokker-Planck equation. In our model, the stationary distribution displays significant reactions to changes in the marginal cost of investment and the strength of the principal's pollution's concern. The mean of the ambient pollution decreases when the production technology becomes cleaner, the efficiency of pollution abatement improves, or the principal is more concerned with the pollution generated by her own firm. Lastly, we show the welfare of both the principal and the agent improves when contracts penalize for pollution. For this, we derive the optimal contract of a misperceiving principal who mistakenly believes pollution does not impact production. Hence this misperceiving principal utilizes a contract which only rewards for financial performance. We then show that the welfare of both the agent and the principal improves if the principal becomes cognizant of the impact of pollution on productivity and the contract rewarding for environmental performance is used instead. In a calibrated version of the model to the U.S. economy, we show that when all firms contract on their own pollution, the reduction in ambient pollution is substantial, amounting to 38.4% less than the ambient pollution when they all ignore the impact of pollution on productivity.

The paper is related to several branches of the literature. First, it connects directly with the literature on dynamic contracts. The seminal work of Holmstrom and Milgrom (1987) derives the optimal contract in a dynamic principal-agent model with CARA agent and shows the relevance of linear incentives for managerial decision-making and principal welfare. Extensions establish the robustness of linear contracts in various models with CARA utility, e.g., Schättler and Sung (1993), Sung (1995), He (2011), Williams (2015), and generalize to non-linear contracts in frameworks with agency, e.g., DeMarzo and Sannikov (2006), Sannikov (2008), and Cvitanić et al. (2009). Our study utilizes the tractability of the CARA utility documented in this literature, but combines it with another payoff-relevant variable: firm pollution, so that the firm's local pollution intensity becomes the unique state variable for the principal's optimal contracting problem. We also integrate the individual firm's contracting problem into an economy with a continuum of firms, examining the aggregate impact of the optimal contract.

Second, the paper connects with the empirical literature dealing with pollution and contracting. Campbell et al. (2007), using data from the Investor Responsibility Resource Center, show that executive compensation is positively related to environmental risk, as measured by the environmental performance of the firm relative to the industry's best practice. Deng and Gao (2013) find a premium in CEO compensation for quality of life, as measured by the Morgan Quitno index constructed from 43 factors, including hazardous waste sites and weather. This premium is significant at the 1% level and amounts to as much as 12% when comparing the 10 most livable to the 10 least livable states in the US. Recent studies examine more specifically the impact of air pollution on executive compensation, e.g., Zhang et al. (2021), Yang et al. (2022), Chan et al. (2022), and Yu et al. (2022), focusing on China. The first three studies find a positive association between senior executive pay and the air quality index (AQI) or the associated air pollution ranking (APR), significant at the 1% level.[7] The last one finds a negative association between senior executive pay sensitivity with respect to corporate performance and the AQI, significant at the 1% level. It also provides evidence of substitution towards other compensation mechanisms such as health insurance and pollution-related cash benefits. Our study provides a theoretical complement to this literature. It shows it is optimal to contract on environmental performance in addition to financial performance, and examines the impact of self-pollution and of model parameters on contractual components, decision-making and aggregate pollution.

Third, the paper relates to a growing empirical literature documenting the impact of pollution on productivity. Graff Zivin and Neidell (2012) show that rising concentrations of ozone in the atmosphere have a significant negative impact on the productivity of agricultural workers; see also Chang et al. (2016). Subsequent studies document mostly negative effects on productivity in a variety of industries and activities, including professional sports (Lichter et al. (2017), Archsmith et al. (2018)), garments and textiles manufacturing (Adhvaryu et al. (2022), He et al. (2019)), call center services (Chang et al. (2019)), government services (Kahn and Li (2020)) and prison factories (Chen and Zhang (2021)). The last study, in particular, documents a non-linear relation between productivity and pollution.[8] Our paper relies on this evidence to model productivity as a non-linear function of pollution. Using country-level

[7] The AQI increases when pollution increases.

[8] Productivity effects are distinct from labor supply effects; for the latter see, e.g., Hanna and Oliva (2015).

data on the economic cost of pollution and CO2 emissions, we fit the pollution impact on productivity by a linear-quadratic function, and adopt this specification in our numerical study.

Lastly, it relates to an empirical strand showing an impact of pollution on agents' preferences. A plethora of studies have documented the detrimental effects of pollution on health and morbidity; see the review by Fuller et al. (2022).[9] Since the health status of an individual affects their welfare (Grossman (1972)), pollution has an indirect effect on preferences through it. A few studies have sought to quantify that effect. Ambrey et al. (2014), in particular, use a life satisfaction approach to estimate the marginal rate of substitution between pollution, as measured by the PM10 concentration, and income, i.e., the willingness-to-pay. Based on their sample, they find households are negatively affected by concentration exceedances and willing to pay roughly 10% of their yearly income to reduce their average exposure to excessive concentrations, i.e., above national guidelines, by one day. Drawing on this literature, we account for an impact of pollution on the agent's and the principal's preferences, and examine implications for optimal contracting and decision-making.

The paper is organized as follows. Section 1 describes the model. Section 2 derives the equilibrium strategy of the agent and the optimal contract in the stationary equilibrium. Section 3 calibrates the model to the U.S. economy and conducts a numerical study to examine properties of equilibrium. Section 5 examines welfare properties of contracts. Conclusions follow. Proofs are in the Appendix.

1 Model

1.1 Individual firm

Before considering equilibrium with a continuum of firms, we focus on the contracting problem for a representative firm. To ease notation, we suppress the index that distinguishes among firms. Such an index will be introduced later to describe the stationary equilibrium among a continuum of firms.

[9] See also report from the Environmental Protection Agency.

Cash flow and environment

A representative firm combines labor and capital to produce. The production per unit of time (in monetary unit) is

$$p_t(\mu + e_t)K, \tag{1}$$

where p is a productivity factor, K is the capital stock, which also serves as a proxy for the physical size of the firm, e_t is the effort exerted by the agent, and μ is the labor provided by the remaining employees of the firm. The agent represents the CEO (or the managerial team) making strategic decisions for the firm and choosing his own effort. Other employees are non decision-makers: they provide labor at the constant rate μ. The total labor at time t is $\mu + e_t$.

The factor p_t in (1) represents productivity per unit of capital-labor at time t. It decomposes as

$$p_t = A\big(1 - D(x_t, \overline{x}_t)\big), \tag{2}$$

for a constant productivity parameter A, which represents the top line productivity in the absence of pollution impact, and a damage function D. The damage function depends on two components: the local pollution generated by the firm and the background pollution generated by all firms operating in the same geographical area. Local firm pollution is measured by its cumulative emissions X_t, e.g., the amount of CO2 and PM2.5 emitted in the air within the firm's plants and buildings or waste water and hazardous material released or leaked on the firm property and its vicinity. Local pollution has a direct effect on working conditions within the firm, hence productivity. Local pollution per unit capital (firm size) is $x = X/K$. We call x the *local pollution intensity*. Background pollution results from the activities of all firms in the geographical area concerned. Denote by $\overline{X}$ the average pollution per firm and by $\overline{K}$ the average firm size. Background pollution per unit capital is $\overline{x} = \overline{X}/\overline{K}$, and this measure captures the spillover effect of background pollution on any individual firm. We call $\overline{x}$ the *background pollution intensity*.[10] When background pollution increases, the environment throughout the geographical area deteriorates, which affects the productivity of all firms operating therein. Therefore we assume that the damage function is an increasing function of both the local pollution intensity and the background pollution intensity. The decomposition in (2) is a stylized version of the damage function found in

[10] Note that the pollution intensities could also be calculated based on the relevant volume of air above the areas concerned. This amounts to a re-scaling of the intensity measures defined.

integrated assessment models (IAMs) of the climate; see Weyant (2017).

The agent of a firm can devote resources to reduce its own pollution. These are running expenses incurred for maintaining waste recycling systems, cleaning up leaks and spills, properly disposing of hazardous materials, maintaining air filters and carbon capture equipment, or monitoring plant and equipment conditions. Let i be investment per unit capital. To reduce firm emission at a rate of $\rho i K$ per unit time, the agent must invest and incur costs at a rate of $iK + k(i, K)$ per unit time, where

$$k(i, K) = \tfrac{\theta^i}{2} i^2 K \tag{3}$$

is a convex cost function, i.e., we assume decreasing returns to scale. The constants ρ and θ^i capture the efficiency of investment in controlling firm emission: efficiency increases with ρ and decreases with θ^i.

Given the agent's effort e and investment i, the firm's cumulative cash flow Y evolves as

$$dY_t = \big(p_t(\mu + e_t)K - i_t K - k(i_t, K)\big)dt + \sigma^y K dB^y_t, \tag{4}$$

where B^y is a standard Brownian motion describing idiosyncratic cash flow risk and $\sigma^y K$ is the cash flow volatility. Local pollution X, generated by the firm, has dynamics

$$dX_t = \big(\lambda p_t(\mu + e_t)K - \rho i_t K\big)dt + \sigma^x K dB^x_t, \tag{5}$$

where B^x is a standard Brownian motion, independent of B^y, describing idiosyncratic pollution shocks, and $\sigma^x K$ is the pollution volatility. The constant λ represents the rate of emissions per unit of production: emissions are proportional to production. Background pollution $\overline{X}$ is an average of local pollution measures and is described in details in Section 1.2.

Contracting problem

The agent receives compensation for managing the firm. He exerts effort e, chooses investment i, and also privately consumes and saves. His effort is costly with cost function (in monetary unit)

$$g(e, x, \overline{x}, K) = \tfrac{1}{2}\theta^e\big(x, \overline{x}\big)e^2\, K, \tag{6}$$

where θ^e is an increasing function in the local and background pollution intensity x and $\overline{x}$ due to the adverse impact of hazardous conditions on safety and health. Savings are invested in a riskless money market account paying interest at a constant rate r.

Given a cumulative compensation I and the background pollution intensity $\overline{x}$, the agent chooses effort e, investment i, and consumption c to maximize the expected utility of consumption

$$\sup_{e,i,c} \mathbb{E}\Big[\int_0^\infty e^{-rt} u(c_t, x_t, \overline{x}_t) dt\Big], \tag{7}$$

subject to the dynamic budget constraint

$$dW_t = (rW_t - g(e_t, x_t, \overline{x}_t, K) - c_t)\, dt + dI_t, \quad W_0 = 0. \tag{8}$$

Here W is the agent's private wealth, i.e., the savings account balance. The agent deposits his instantaneous compensation dI into it and withdraws cash to support his consumption choice c. To simplify the presentation, the agent's subjective discount rate is set equal to r. The agent's initial wealth is also normalized to be zero.

The agent's utility function is

$$u\,(c, x, \overline{x}) = -\exp\big(-\gamma_c c + \gamma_x(\ell x + (1-\ell)\overline{x})\big). \tag{9}$$

It depends on consumption, local firm pollution intensity and background pollution intensity. Both types of pollution have a negative impact on health, therefore utility. Here, the positive constant ℓ represents the weight of the local pollution intensity, $1 - \ell$ the weight of the background pollution intensity in forming the index of pollution $\ell x + (1 - \ell)\overline{x}$ the agent is exposed to. These weights can be thought of as percentage of time the agent spends in and outside firm, where the agent suffers from the local and background pollution, respectively. The agent's absolute aversion to consumption risk is γ_c and the absolute aversion to pollution risk is $\gamma_x \geq 0$. When $\gamma_x = 0$, the agent is purely consumption-driven and is not sensitive to environmental pollution. The effect of pollution on the agent's utility is per unit capital because the agent operates in a unit area during each small time interval.

The agent observes cash flow and local pollution shocks B^y and B^x, but the principal only observes the cash flow Y and the local pollution level X. The principal is therefore unable to observe or contract on the agent's effort e and investment i. This creates a moral hazard problem for the firm. Both the agent and the principal observe the background pollution $\overline{X}$

in the geographical area and the firm size K.

The principal has incentives to contract on the local pollution (cumulative emissions) X and the cash flow Y of her own firm in order to improve her welfare. In contrast, she has no incentive to contract on background pollution $\overline{X}$, because there is a continuum of firms and any single firm's emissions are infinitesimal relative to the aggregate $\overline{X}$. It follows that each firm takes $\overline{X}$ as given and does not consider its own impact on $\overline{X}$ in making firm decisions.

The principal is risk neutral. She chooses the compensation process I, adapted to the filtration generated by X, $\overline{X}$, and Y, so as to maximize the present value of future profits net of the compensation to the agent and the environmental cost

$$\sup_I \mathbb{E}\left[\int_0^\infty e^{-rt}\left(dY_t - dI_t - mK\big(\zeta x_t + (1-\zeta)\overline{x}_t\big)\,dt\right)\right], \tag{10}$$

Maximization is subject to the agent's participation constraint which stipulates that the value function of the agent's optimization problem (7) be at least the reservation utility R, the agent's incentive compatibility constraint that X and Y be determined by the agent's optimal actions (c^*, e^*, i^*), and the principal's participation constraint which requires the value in (10) be at least R^p. In (10), $mK(\zeta x_t + (1-\zeta)\overline{x}_t)$ is the cost to the principal from all sources of pollution, aggregated from both local and background pollution with respective weights ζ and $1-\zeta$, and m is a non-negative constant quantifying her sensitivity to environmental pollution. This cost can be interpreted as a direct health-related cost (to the principal) from pollution, as a concern for the welfare of her firm's employees, or as a concern for the environment and the welfare of the population. The cost is proportional to firm size K because the principal's exposure is proportional to the total amount of pollution on the firm's property.

1.2 Equilibrium

We now consider a continuum of firms indexed by $n \in [0, 1]$. Given the background pollution level, each firm faces the contracting problem described in the previous section. We introduce a superscript n to indicate firm specific quantities. For example, X^n is the local pollution amount from firm n when the agent employs the optimal strategy (c^{n*}, e^{n*}, i^{n*}) incentivized by the optimal contract. The capital stock of this firm, a proxy for the firm's size, is K^n. Given the background pollution $\overline{X}$, we assume that cash flow shocks and pollution shocks are idiosyncratic in each firm, i.e., $\{B^{y,n}\}_n$ and $\{B^{x,n}\}_n$ are two continua of mutually independent standard Brownian motions.

To illustrate the derivation of background pollution, let us first consider a regional economy with N firms. We assume that pollution created over different parts of the region spreads and blends together to form background pollution affecting the whole region. This background pollution intensity is the ratio between total pollution and total capital stock

$$\overline{x} = \frac{\sum_{n=1}^{N} X^n}{\sum_{n=1}^{N} K^n}.$$

Assume that $\{X^n, K^n\}$ are i.i.d. realizations of a pair of random variables (X, K). We obtain from the law of large number that

$$\frac{\sum_{n=1}^{N} X^n}{\sum_{n=1}^{N} K^n} = \frac{\frac{1}{N}\sum_{n=1}^{N} X^n}{\frac{1}{N}\sum_{n=1}^{N} K^n} \to \frac{\mathbb{E}[X]}{\mathbb{E}[K]}, \quad \text{as } N \to \infty,$$

This motivates us to introduce the background emission intensity measure

$$\overline{x}_t = \frac{\mathbb{E}[X_t]}{\mathbb{E}[K]}, \qquad \text{for } t \geq 0. \tag{11}$$

We focus on a stationary equilibrium in which the background emission intensity is time independent, i.e., $\overline{x}_t \equiv \overline{x}$, for a constant $\overline{x}$.

We now draw on the notion of mean-field game to define equilibrium with a continuum of firms.

Definition 1 *An equilibrium is a collection* $(c^*, e^*, i^*, I^*, X, Y, \overline{x})$ *such that*

(i) Given $\overline{x}$*, for a representative firm of size* K*,* I^* *is the optimal contract for the principal's problem* (10)*,* (c^*, e^*, i^*) *is the agent's optimal strategy for* (7)*, and* X, Y *are the associated local firm pollution and cash flow processes;*

(ii) $\overline{x}$ *is the background pollution intensity, given by*

$$\overline{x} = \frac{\mathbb{E}\big[X_t\big]}{\mathbb{E}[K]}, \qquad \textit{for all } t \geq 0.$$

2 Optimal contract and stationary distribution

For a given background pollution intensity $\overline{x}$ and a firm size K, we first characterize the optimal contract for a representative firm. Then we examine the equilibrium and the stationary density. Throughout this section, K is assumed to be a positive constant.

2.1 Agent's optimal strategies

For a given background pollution intensity $\overline{x}$, which may be time-dependent, a firm size K, and a contract I, we first solve the agent's problem (7) and derive the agent's optimal strategies for a representative firm.

Define the agent's continuation value as

$$U_t^a = \sup_{e,i,c} \mathbb{E}_t \Big[\int_t^{\infty} e^{-r(s-t)} u(c_s, x_s, \overline{x}_s) ds \Big],$$

subject to (8). Due to the agent's CARA utility function, we introduce $\mathcal{U}$ via

$$U_t^a = -e^{-\gamma_c r(W_t + \mathcal{U}_t)}, \tag{12}$$

so that $\mathcal{U}$ is the agent's (normalized) certainty equivalent when he starts with zero initial wealth. The martingale representation theorem ensures that $\mathcal{U}$ admits the decomposition

$$d\mathcal{U}_t = dH_t + Z_t^y dY_t + Z_t^x dX_t, \tag{13}$$

for two processes Z^x and Z^y and a finite variation process H. We call Z^x the contract sensitivity with respect to the local pollution and Z^y the contract sensitivity with respect to the firm cash flow.[11] The process H and the agent's optimal strategies are to be determined by the dynamic programming principle. The following result summarizes the dynamics of the continuation certainty equivalent and the agent's optimal strategies.

Lemma 2 *For a given background pollution intensity $\overline{x}$, a firm size K, and a compensation*

[11] Equivalently, the sensitivities of the agent's continuation utility with respect to the local pollution and the firm cash flow are $-r\gamma_c U_t^a Z^x$ and $-r\gamma_c U_t^a Z^y$, respectively.

stream I, the agent's continuation certainty equivalent follows

$$d\mathcal{U}_t = \left\{ r\mathcal{U}_t + \frac{\gamma_x}{\gamma_c}\big(\ell x_t + (1-\ell)\overline{x}_t\big) + \Phi(e_t^*, Z_t^x, Z_t^y) \right\} dt + Z_t^x K \sigma^x dB_t^x + Z_t^y K \sigma^y dB_t^y - dI_t \quad (14)$$

$$\Phi(e_t^*, Z_t^x, Z_t^y) = -\frac{1}{\gamma_c}\log r + g(e_t^*, x_t, \overline{x}_t, K) + \frac{\gamma_c r}{2} K^2 \big[(Z_t^y \sigma^y)^2 + (Z_t^x \sigma^x)^2\big]. \quad (15)$$

The agent's optimal consumption, effort, and environmental investment strategies are

$$c_t^* = -\frac{1}{\gamma_c}\log r + \frac{\gamma_x}{\gamma_c}\big(\ell x_t + (1-\ell)\overline{x}_t\big) + rW_t + r\mathcal{U}_t, \quad (16)$$

$$e_t^* = Z^e \frac{p(x_t, \overline{x}_t)}{\theta^e(x_t, \overline{x}_t)}, \quad (17)$$

$$i_t^* = \begin{cases} \frac{-\rho Z_t^i - 1}{\theta^i}, & Z_t^i < -\frac{1}{\rho} \\ 0, & \text{otherwise} \end{cases} \quad (18)$$

where $Z^e := Z^y + \lambda Z^x$, $Z^i := \frac{Z^x}{Z^y}$, and $x_t = X_t/K$ with X following (5) *with agent's strategies e^* and i^*.*

It follows from (14) and the transversality condition $\lim_{T\to\infty}\mathbb{E}[e^{-rT}\mathcal{U}_T] = 0$ that $\mathcal{U}$ has the Feynman-Kac representation

$$\mathcal{U}_t = \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}\left[-\left(\Phi(e_s^*, Z_s^x, Z_s^y) + \frac{\gamma_x}{\gamma_c}\big(\ell x_t + (1-\ell)\overline{x}\big)\right) ds + dI_s\right]\right]. \quad (19)$$

where $\Phi(e_s^*, Z_s^x, Z_s^y)$ is defined in (15). Therefore, the agent's certainty equivalent summarizes the discounted future compensation net the negative pollution impact $\frac{\gamma_x}{\gamma_c}(\ell x + (1-\ell)\overline{x})$, the effort cost $g(e^*, x, \overline{x}, K)$, and the risk aversion cost due to the instantaneous variance $K^2[(Z^y\sigma^y)^2 + (Z^x\sigma^x)^2]$ of the certainty equivalent.

In (14), Z^x and Z^y are the agent's certainty equivalent volatility coefficients. As will be shown, they are also the contract sensitivities with respect to the local pollution amount and the firm cash flow. Both contract sensitivities are to be determined by the principal in the optimal contract. Because an adverse environment hurts productivity and reduces the principal's utility, it is intuitive that the principal would penalize the agent for high pollution X by choosing a negative contract sensitivity Z^x. For given Z^x and Z^y, the agent's optimal effort is determined by

$$\max_{e} \Big\{ (Z^y + \lambda Z^x)(\mu + e)p(x, \overline{x}) - g(e, x, \overline{x}) \Big\},$$

where $Z^e := Z^y + \lambda Z^x$ is the marginal benefit for each additional dollar of cash flow net of the marginal environmental cost. For each additional dollar of firm cash flow, the agent's continuation certainty equivalent increases by Z^y, meanwhile the pollution increases by λ, hence the agent is penalized by $-\lambda Z^x$ with $Z^x < 0$. We call Z^e the *gross contract sensitivity to cash flow*. The optimal effort e^* in (17) is determined by matching the marginal benefit of effort and its marginal cost.

The optimal environmental investment is determined by

$$\max_{i} \Big\{ - Z^x \rho i - Z^y \big(i + k(i, K)\big) \Big\},$$

where the first term in the maximization problem is the benefit of environmental improvement via investment and the second term represents the cost of investment. For i dollars invested, emissions are reduced by ρi, hence the agent's certainty equivalent is improved by $-Z^x \rho i$. Meanwhile, the cost of investment is $i + k(i, K)$, which reduces the firm's cash flow, hence the agent's certainty equivalent by $Z^y(i + k(i, K))$. The optimal environmental investment i^* is obtained by matching the marginal benefit of investment and the marginal cost. Equation (18) shows that the optimal investment is determined by the ratio $Z^i := Z^x/Z^y$, which is the relative contract sensitivity between the endogenous environmental variable and the firm cash flow. Only when the relative contract sensitivity becomes sufficient negative (less than $-1/\rho$) does a positive investment become optimal. When $Z^i \geq -1/\rho$, the marginal benefit of investment is not sufficient to offset the marginal cost of cash flow reduction. As a result, environmental investment is sub-optimal. The threshold $-1/\rho$ increases with the efficiency parameter ρ of investment, all other things equal, implying a smaller magnitude of the contract sensitivity Z^x is required to incentivize environmental investment.

The agent's optimal consumption in (16) is determined by matching his marginal utility of consumption to the marginal value of wealth, $u'(c^*, x, \overline{x}) = \partial_W U^a$. The agent's continuation certainty equivalent satisfies (14). It increases with the costs from local and background pollution $\frac{\gamma_x}{\gamma_c}\big(\ell x + (1-\ell)\overline{x}\big)$, the agent's effort cost g and the risk aversion cost $\frac{\gamma_c r}{2} K^2 \big[(Z^y \sigma^y)^2 + (Z^s \sigma^x)^2\big]$, with the latter two costs encapsulated in Φ. Combining (8), (14), (15) and (16), we obtain that

$$U_t^a = U_0^a\,\mathcal{E}\Big(-\gamma_c r K\int_0^t Z_s^x\sigma^x dB_s^x + Z_s^y\sigma^y dB_s^y\Big),$$ [12]

Therefore the continuation utility is a martingale under the optimal consumption strategy, a consequence of the agent's private saving and his CARA utility, see also He (2011). The exponential structure of CARA utility implies that the agent's marginal utility is proportional to its continuation utility. If marginal utility were not a martingale, then the agent would benefit by saving and postponing consumption to a later time when marginal utility is higher on average. The agent's private saving therefore equalizes (expected) marginal utility across time, thereby ensuring it is a martingale.

2.2 The optimal contract

After deriving the agent's optimal strategies for a given contract, we examine the optimal contract of a representative firm in this section. The principal chooses the contract sensitivities Z^e and Z^i to optimize her welfare (10) subject to agent's participation and incentive compatibility constraints. We assume that the background pollution intensity $\overline{x}$ is a constant throughout the remaining of this section, because we will search for a stationary equilibrium later on.

Define the principal's value function as

$$U_t^p = \sup_{Z^e,Z^i}\mathbb{E}_t\Big[\int_t^\infty e^{-r(s-t)}\Big(dY_s - dI_s - mK\big(\zeta x_s + (1-\zeta)\overline{x}\big)ds\Big)\Big]. \tag{20}$$

Introduce firm's total value V via

$$V_t = U_t^p + \mathcal{U}_t, \tag{21}$$

which is the sum of principal's value and the agent's certainty equivalent. Combining (19) and (20), we obtain a representation for the total value

$$\begin{aligned} V_t = \sup_{Z^e,Z^i}\mathbb{E}_t\Big[\int_t^\infty e^{-r(s-t)}\Big\{&p(x_s,\overline{x}_s)(\mu+e_s^*)K - i_s^*K - k(i_s^*,K) - \Phi(e_s^*,Z_s^x,Z_s^y)\\ &-\Big(\frac{\gamma_x}{\gamma_c}\ell + mK\zeta\Big)x_s - \Big(\frac{\gamma_x}{\gamma_c}(1-\ell)+mK(1-\zeta)\Big)\overline{x}\Big\}ds\Big], \end{aligned} \tag{22}$$

[12]Here $\mathcal{E}\left(-\int_0^t\theta_s dB_s\right) = \exp\left(-\frac{1}{2}\int_0^t|\theta_s|^2ds - \int_0^t\theta_s dB_s\right)$ is an exponential martingale.

which is the expected discounted value of cash flow net of the investment cost, the effort and risk costs Φ to the agent, and the local and background pollution costs to both the agent and the principal. Consider the agent's certainty equivalent $\mathcal{U}$ as the principal's state variable, then the principal's value is $U^p = V - \mathcal{U}$. The optimal contract sensitivities Z^e and Z^i are determined by the problem (22).

Given the background pollution intensity $\overline{x} = \overline{X}/\overline{K}$, taking the endogenous variable $x = X/K$ as the state variable for the principal's problem (22), we obtain the following characterization for the value V and the optimal contract.

Proposition 3 *The value function V satisfies the following HJB equation*

$$rV = \frac{1}{2}(\sigma^x)^2\,\partial^2_{xx}V + H(x,\overline{x},\partial_x V), \tag{23}$$

where the Hamiltonian is

$$\begin{aligned} H(x,\overline{x},\xi) = \max_{Z^e,Z^i}\Big\{ &\big[\lambda p(x,\overline{x})(\mu+e^*) - \rho i^*\big]\xi \\ &+ p(x,\overline{x})(\mu+e^*)K - i^*K - k(i^*,K) - \Phi(e^*,Z^x,Z^y) \\ &- \Big(\frac{\gamma_x}{\gamma_c}\ell + mK\zeta\Big)x - \Big(\frac{\gamma_x}{\gamma_c}(1-\ell) + mK(1-\zeta)\Big)\overline{x}\Big\}, \end{aligned} \tag{24}$$

with (e^*, i^*) given in (17) and (18), Φ given in (15), $Z^e = Z^y + \lambda Z^x$, and $Z^i = \frac{Z^x}{Z^y}$. The optimal contract sensitivities Z^{e*} and Z^{i*} are given by the optimizers of the Hamiltonian. The optimal contract sensitivities to firm cash flow and local pollution are denoted by Z^{y*} and Z^{x*}, respectively. They are functions of the state variable x.

An optimal compensation stream is

$$\begin{aligned} dI^*_t =\Big[&-\frac{1}{\gamma_c}\log r + r\mathcal{U}_0 + g(e^*_t,x_t,\overline{x},K) + \frac{\gamma_x}{\gamma_c}\big(\ell x_t + (1-\ell)\overline{x}\big) + r\Psi_t \\ &+ r\int_0^t Z^{y*}_s dY_s + r\int_0^t Z^{x*}_s dX_s\Big]dt, \end{aligned} \tag{25}$$

where $\mathcal{U}_0$ is the agent's reservation certainty equivalent, Y and X follow (4) *and* (5) *with* *the agent's optimal effort e^* and investment i^* therein, and*

$$\Psi_t = \int_0^t \Big[\frac{\gamma_c r}{2} K^2\big((Z_s^{y*}\sigma^y)^2 + (Z_s^{x*}\sigma^x)^2\big) \\ - Z_s^{y*}\big(p_s(\mu + e_s^*)K - i_s^*K - k(i_s^*, K)\big) - Z_s^{x*}\big(\lambda p_s(\mu + e_s^*)K - \rho i_s^* K\big)\Big] ds.$$

Receiving this compensation stream, the agent optimally chooses not to save and to consume all his compensation net of his effort cost.

The optimal compensation stream depends on the observables $x, \overline{x}$, and Y. It rebates the agent's effort cost $g(e^*, x, \overline{x}, K)$ and compensates for the environmental impact $\frac{\gamma_x}{\gamma_c}(\ell x + (1-\ell)\overline{x})$ on the agent. The Ψ term compensates the agent for risk bearing net expected compensations depending on firm cash flow and firm pollution. In the optimal compensation, the principal contracts on both firm cash flow Y and firm local pollution X. Because the optimal contract sensitivities are state-dependent, the optimal compensation depends on the full history of Y and X via $\int_0^t Z_s^{y*} dY_s$ and $\int_0^t Z_s^{x*} dX_s$.

To better understand the optimal contract sensitivities, let us first consider the case where neither the productivity nor the effort cost depend on the pollution intensities x and $\overline{x}$. In this case, the value function V is a linear function of x. The optimal contract sensitivities are constants.

Corollary 4 *When $p \equiv p_0$ and $\theta^e \equiv \theta_0^e$ for constants p_0 and θ_0^e, the optimal contract sensitivities are constants and they are the maximizer of*

$$\max_{Z^e, Z^i} \Big\{ p_0(\mu + e^*)K - \frac{\theta_0^e}{2}(e^*)^2 K - \frac{1}{r}\Big(\frac{\gamma_x}{\gamma_c}\ell + mK\zeta\Big)\big(\lambda p_0(\mu + e^*) - \rho i^*\big) \\ - i^*K - \frac{\theta^i}{2}(i^*)^2 K - \frac{\gamma_c r}{2}(Z^e)^2 K^2 \frac{(\sigma^y)^2 + (\sigma^x Z^i)^2}{(1 + \lambda Z^i)^2}\Big\}, \tag{26}$$

where (e^*, i^*) are given in (17) and (18). The principal's optimal value is

$$U^P(\mathcal{U}, x) = -\mathcal{U} - \frac{1}{r}\Big(\frac{\gamma_x}{\gamma_c}\ell + mK\zeta\Big)x + \textit{constant}, \ \textit{where}, \tag{27}$$

$$\textit{constant} = \frac{1}{r\gamma_c}\log r - \left(\frac{\gamma_x}{\gamma_c}(1-\ell) + mK(1-\zeta)\right)\frac{\ell}{r}\overline{x} + \frac{\textit{maximum value of } (26)}{r}. \tag{28}$$

To shed light on the optimal contract sensitivities when the productivity and effort cost are independent of pollution, let us consider two special cases: (i) the limiting case where the agent is risk neutral, i.e., $\gamma_c \to 0$ and (ii) the case where the pollution reduction is too inefficient to induce an environmental investment. In the first case, $\gamma_c \to 0$, but we assume γ_x/γ_c is a constant, therefore γ_x converges to zero as well. In this case, the maximization in (26) reduces to

$$\max_{e^*,i^*}\Big\{p_0(\mu+e^*)K-\frac{\theta^e}{2}(e^*)^2K-\frac{1}{r}\Big(\frac{\gamma_x}{\gamma_c}\ell+mK\zeta\Big)\big(\lambda p_0(\mu+e^*)-\rho i^*\big)-i^*K-\frac{\theta^i}{2}(i^*)^2K\Big\}. \quad (29)$$

When

$$\frac{\lambda}{r}\left(\frac{\gamma_x}{\gamma_c}\frac{\ell}{K}+m\zeta\right)<1 \quad \text{and} \quad \frac{\rho}{r}\left(\frac{\gamma_x}{\gamma_c}\frac{\ell}{K}+m\zeta\right)>1, \quad (30)$$

the agent's optimal effort and optimal investment strategy

$$e^*=\frac{p_0}{\theta_0}\Big(1-\frac{\lambda}{r}\Big(\frac{\gamma_x}{\gamma_c}\frac{\ell}{K}+m\zeta\Big)\Big) \quad \text{and} \quad i^*=\frac{1}{\theta^i}\Big(\frac{\rho}{r}\Big(\frac{\gamma_x}{\gamma_c}\frac{\ell}{K}+m\zeta\Big)-1\Big)$$

are both positive. The optimal contract sensitivities to firm cash flow and local pollution are

$$Z^{y*}=1 \quad \text{and} \quad Z^{x*}=-\frac{1}{r}\Big(\frac{\gamma_x}{\gamma_c}\frac{\ell}{K}+m\zeta\Big).$$

The unit contract sensitivity to cash flow means that the principal delegates the firm to the agent. Meanwhile, the principal penalizes the agent more severely for local pollution when either the agent or the principal have greater environmental concern (γ_x, m, ℓ or ζ increases).

To understand the first condition in (30), consider an unit increase in the agent's effort. It increases the firm's production by p_0K, and the local pollution intensity by λp_0, which leads to an incremental environmental cost to the principal equal to $\frac{\lambda}{r}(\frac{\gamma_x}{\gamma_c}\frac{\ell}{K}+m\zeta)p_0K$, due to the linear form of the principal's value function in x (see (27)). The first condition in (30) ensures that the marginal benefit of effort outweighs the marginal environmental cost of effort, so that the optimal effort is positive. For the second condition in (30), consider a unit investment in emission reduction. It reduces the local pollution intensity by ρ, hence introduces an incremental environmental benefit given by $\frac{\rho}{r}(\frac{\gamma_x}{\gamma_c}\frac{\ell}{K}+m\zeta)K$, again due to the linear form of the principal's value function in x. Therefore the second condition in (30) implies that the marginal benefit of environmental investment outweighs the direct marginal cost of investment, hence ensuring a positive optimal environmental investment.

Let us now consider another special case of Corollary 4, where the first condition in (30) holds, but the second condition there fails, and the agent is risk averse. In this case, the optimal environmental investment and the contract sensitivity to pollution are both zero, and the optimization in (26) is reduced to

$$\max_{Z^y}\left\{\left[1-\frac{\lambda}{r}\Big(\frac{\gamma_x}{\gamma_c}\frac{\ell}{K}+m\zeta\Big)\right]p_0\Big(\mu+\frac{p_0}{\theta_0^e}Z^y\Big)K-\frac{1}{2\theta_0^e}\big(p_0Z^y\big)^2K-\frac{\gamma_c r}{2}\big(\sigma^yKZ^y\big)^2\right\}.$$

The optimal contract sensitivity to firm cash flow is

$$Z^{y*}=\frac{1-\frac{\lambda}{r}\Big(\frac{\gamma_x}{\gamma_c}\frac{\ell}{K}+m\zeta\Big)}{1+\frac{\gamma_c r\theta_0^eK(\sigma^y)^2}{p_0^2}}. \tag{31}$$

Comparing to the optimal contract sensitivity $\big(1+\gamma_c r\theta_0^eK(\sigma^y)^2/p_0^2\big)^{-1}$ in the benchmark model of Holmstrom and Milgrom (1987) with private savings He (2011), the contract sensitivity in (31) is smaller due to the environmental concerns of the agent and the principal.

When either p or θ^e depend on x, (23) no longer admits an explicit solution, and the optimal contract sensitivities are state-dependent as well. We solve this equation numerically to obtain the optimal contract in Section 3. To facilitate the numerical procedure, we impose two asymptotic boundary conditions for (23) when x is zero or large. When $x=0$, we impose a Neumann boundary condition $V'(0)=-\Big(\frac{\gamma_x}{\gamma_c}+mK\zeta\Big)$. At the other extreme, we assume that either $\lim_{x\uparrow\infty}p(x,\overline{x})=0$ or $\lim_{x\uparrow\infty}\theta^e(x,\overline{x})=\infty$. As a result, the environment becomes so polluted that either productivity vanishes or the agent's marginal cost of effort explodes. In either case, the agent's optimal effort e^* and the optimal contract sensitivity to cash flow Z^y are both null. When x is sufficiently large, we therefore impose the asymptotic boundary condition by setting $e^*=Z^y=0$ in (23) and (24).

2.3 Equilibrium and the stationary distribution

After the optimal contract is obtained for a representative firm, we consider a continuum of firms with homogeneous constant size K. When the principal of an individual firm uses the optimal contract to incentivize the agent's optimal effort e^* and the optimal environmental investment i^*, the endogenous local pollution intensity follows

$$dx_t=\big(\lambda p(x_t,\overline{x}_t)(\mu+e_t^*)-\rho i_t^*\big)dt+\sigma^x dB_t^x.$$

Denote the density of x as $m(t,x)$. It follows from Achdou et al. (2022) that m satisfies the Kolmogorov-Fokker-Planck equation

$$\partial_t m(t,x) - \frac{1}{2}(\sigma^x)^2 \partial^2_{xx} m(t,x) + \partial_x \Big[\big(\lambda p(x, \overline{x})(\mu + e^*) - \rho i^* \big) m(t,x) \Big] = 0. \tag{32}$$

We look for a stationary equilibrium in which x admits a stationary density $m(x)$ independent of time. As a result, $m(x)$ is a stationary solution to (32), i.e., it satisfies

$$-\frac{1}{2}(\sigma^x)^2 \partial^2_{xx} m(x) + \partial_x \Big[\big(\lambda p(x, \overline{x})(\mu + e^*) - \rho i^* \big) m(t,x) \Big] = 0. \tag{33}$$

Given the stationary density m, and thanks to the common firm size K, the background pollution density $\overline{x} = X/K$ is

$$\overline{x} = \int x \, m(dx). \tag{34}$$

In conclusion, a stationary equilibrium, in Definition 1, is characterized by the HJB equation (23), the Kolmogorov-Fokker-Planck equation (33), and the consistency condition (34).

3 Quantitative model implications

In this section, we first calibrate model parameters, then present the quantitative implications of our model. The model's solution is obtained by numerically solving the system of coupled equations (23), (33), and (34).

3.1 Model calibration

The model parameters are calibrated to several data sources. We focus on the US economy due to data availability. We interpret the cash flow dY_t in (4) as the US annual GDP (in trillion dollars), the capital stock K as the Stock of Fixed Assets and Consumer Durable Goods, which includes, for example, buildings, machines, software, and intellectual property products. We have $K = 85$ trillion dollars in 2021.[13] The emissions amount dX_t in (5) is

[13]Both the GDP and the Fixed Assets data are obtained from the Bureau and Economic Analysis.

measured by the US annual carbon dioxide emissions (in million metric tons).[14] To obtain company average quantities, we normalize dY_t, dX_t, and K by the average number of public firms in the US between 1990 and 2019.[15]. To estimate the volatility parameters σ^y and σ^x, we remove the trend from the ratio of annual GDP over fixed assets and from the ratio of annual carbon emissions over fixed assets, then measure the standard deviation of residuals.[16] Resulting estimates are $\sigma^y = 0.97\%$ and $\sigma^x = 0.98\%$.

To estimate the emissions intensity parameter λ in (5), we use the pre-pandemic GDP and carbon emissions in 2019 to obtain $\lambda = 0.246$ kg CO2/dollar.[17] To estimate the efficiency parameter ρ for the environmental investment, we examine the current commercialized carbon capture technology. The last comprehensive analysis of the technology, conducted by the American Physical Society in 2011, estimated that it would cost 600 dollars to absorb one ton of CO2.[18] This leads to an estimate of $\rho = 1.67$ kg CO2/dollar.

The agency parameter θ^e, the productivity function p and the labor parameter μ are determined in the following way. First, the average ratio of annual GDP over fixed asset is 0.295 between 1990 and 2020. This is approximated by $p^{\text{current}}\mu$ in the model, where p^{current} is the productivity value under the current emissions intensity.[19] To separate p^{current} and μ, we use the empirical evidence in the literature pertaining to the performance sensitivity of the average executive contract. Jensen and Murphy (1990) report a contract performance sensitivity of 0.3% in their sample (1969-1983). Hall and Liebman (1998) document a higher sensitivity around 2.5%. Controlling for the firm risk, Aggarwal and Samwick (1999) report a mean contract sensitivity equal to 6.94%. We choose $p^{\text{current}} = 0.05$ and $\theta^e = 25$ so that the

[14]Carbon dioxide constitutes the majority of the U.S. greenhouse gas emissions, roughly 80% of total greenhouse emissions since 1990. The emissions data is from the Environmental Protection Agency (EPA).

[15]The average number of US public firms between 1990 and 2019 is 5614. Data are from World Bank.

[16]Our data set starts from 1990, the earliest year with emissions data on the EPA website. There is a clear decreasing trend in the ratio of annual emissions to fixed assets due to the increase in energy efficiency over the last three decades.

[17]In the estimation, we ignore the impact of the environmental investment i. In 2022, the US government announced investment plans equal to 2.3 billion to cut U.S. carbon pollution (see Report). If this funding were spent on carbon capture, the current commercialized technology would allow for a reduction of CO2 emissions by roughly 4 million tons, which is a minute fraction of U.S. CO2 emissions equal to 5,259 million tons in 2019.

[18]See the report here. The cost decreases if the most recently developed technology is used, as it pulls a ton of CO2 from the atmosphere at a cost ranging from 94 to 232 dollars (see the Nature report. This technology is yet to fully commercialized. The U.S. Department of Energy goal is to reduce the cost under 100 dollars per ton by 2030 (see report).

[19]As the numerical results show, executive effort e makes a negligible contribution to the GDP .

contract financial performance sensitivity in (31) for the benchmark Holmström–Milgrom model is around 3.5%, in line with this empirical evidence. The associated labor parameter is $\mu = 5.9$.

In order to determine the functional form of the productivity function p, we use the CREA report, which provides the cost of air pollution for different countries. Figure 1 presents the economic cost of air pollution as the share of GDP with respect to the CO2 emissions intensity (kg per dollar GDP based on purchasing power parity (PPP)) for the top 15 countries by GDP. The figure shows that the cost of air pollution stays at roughly the same level when the CO2 emissions intensity is low. However, when the CO2 emissions intensity is high, the economic cost becomes much larger. For instance, the CO2 emissions intensity of China is 0.48 in 2018 and CREA estimates a cost of 6.6% GDP; the emissions intensity of Russia is 0.39 and the cost of air pollution is 4.1% GDP; the emissions intensity of India is 0.27 and the cost of air pollution is 5.4% GDP. We fit the data pertaining to the top 15 countries with the following form of the damage function

$$D(x^{\mathrm{agg}}) = D_0 + D_1(x^{\mathrm{agg}} - x_0)_+^2. \tag{35}$$

When the aggregated pollution intensity x^{agg}, from both the local and background pollution, is below a threshold x_0, the damage is a constant D_0. When x^{agg} exceeds x_0, the damage increases quadratically. The aggregated pollution intensity x^{agg} is assumed to be the weighted average $\xi x + (1-\xi)\overline{x}$ with $\xi = 0.5$. Here we assume that the local pollution intensity and the background pollution intensity perfectly substitute for each other. The equal weight reflects the assumption that employees spend half of their time on the firm property and half of it elsewhere. The resulting productivity function net of damages is

$$p(x, \overline{x}) = A\Big(1 - D\big(0.5(x + \overline{x})\big)\Big),$$

where A is the efficient productivity without damages from pollution. The fit of the data in Figure 1 to the damage function (35) along with the information that the U.S. CO2 emissions intensity is 0.242 (kg per PPP dollar GDP) and $p^{current} = 0.05$, gives the estimated net damage productivity function

$$p(x, \overline{x}) = 0.05 - 0.02\big(0.5(x + \overline{x}) - 0.2\big)_+^2.$$

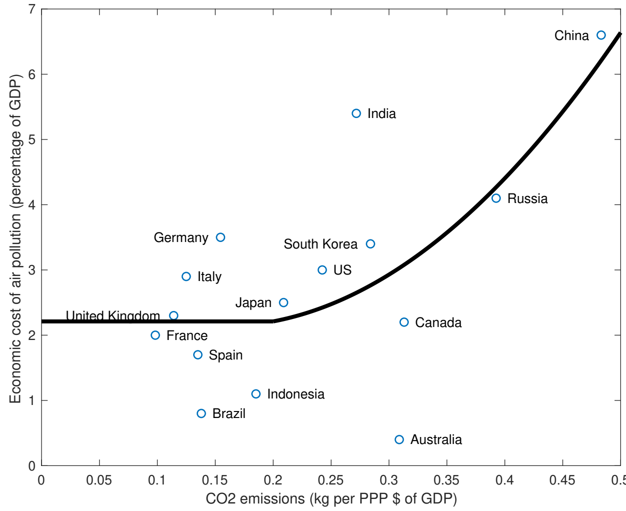

Figure 1: Fitted damage function. The fitted linear-quadratic damage function is $D(x^{\text{agg}}) = 0.02$ when $x^{\text{agg}} < 0.2$; $D(x^{\text{agg}}) = 0.39(x^{\text{agg}} - 0.2)^2 + 0.02$ when $x^{\text{agg}} \geq 0.2$.

Remaining parameters are calibrated as follows. The interest rate $r = 0.05$ is set at a value commensurate with recent and historical experience. The agent's absolute risk aversion is $\gamma^c = 5$, consistent with the median value reported in Haubrich (1994). His aversion to workplace pollution γ^x is set to be zero, so that the agent is only motivated by consumption.[20] The risk neutral principal's sensitivity to environmental pollution is also selected to vary, between $m = 0.04$ and $m = 0.05$. As we will show, variations in this narrow range will produce substantial changes in outcomes. Moreover, in this range of m, the second inequality of (30) fails in all experiments, implying the principal would not invest in emissions reduction if productivity were not decreasing in pollution. Our experiments therefore show the impact of productivity reduction due to pollution. Finally, the investment cost parameter is $\theta^i = 2$ in numerical experiments.

3.2 Optimal policies, contract and pollution distribution

In this section we study the optimal policies, the contract, and the distribution of the local pollution intensity across firms in the calibrated model. We also examine the impact of the

[20]The agent's aversion to workplace pollution γ_x has a much smaller impact than the principal's sensitivity to pollution m, because γ_x is dominated by mK in (24) when K is large

Table 1: **Parameter Values**

This table reports parameter values used for simulations.

Parameter	**Variable**	**Value**
r	Interest rate	0.05
μ	Production labor	5.9
K	Firm size	85
σ^x	Environmental state volatility	0.0098
σ^y	Cash flow volatility	0.0097
γ^x	Agent environmental risk aversion	0
γ^c	Agent consumption risk aversion	5
A	Efficient productivity	0.05
D^q	Damage function quadratic coefficient	0.39
x_0	Environmental threshold	0.2
ℓ, ζ, ξ	Weight on local pollution intensity	0.5
m	Principal environmental concern	0.04
ρ	Efficiency of environmental investment	1.67
λ	Production emission intensity rate	0.246
θ^e	Effort cost coefficient	25
θ^i	Investment cost coefficient	2

pollution rate λ, the efficiency of environmental investment ρ, and the principal's concern for pollution m.

Figure A-1 presents the optimal policies, contract, and the distribution of local pollution intensity across firms in the calibrated model. In panel (a), the black solid lines represent the corresponding quantities when the productivity does not depend on pollution, i.e., $p \equiv A$. In this case, environmental concerns from the principal are not sufficiently strong to induce pollution reducing investment and the contract sensitivity with respect to pollution is also zero. The expected local pollution always grows over time due to the emissions generated by production. As a result, there is no stationary distribution of the local pollution.

The red dotted lines in panel (a) of Figure A-1 show the optimal policies and contract sensitivities when productivity deteriorates with pollution. There exists a threshold x^* (around 0.08) such that when $x < x^*$ the agent's optimal effort and the optimal contract sensitivity to cash flow are flat in x. They are lower than their counterparts in the model with constant productivity. This is because the principal anticipates the future low productivity states due to heavy pollution, hence reduces production, pollution growth, and the probability that the aggregated pollution from both local and background pollution exceeds the threshold x_0

where the firm productivity starts to decrease. When $x < x^*$ both investment and contract sensitivity to pollution are zero. However, when x exceeds the threshold x^*, the investment in pollution reduction becomes positive. As (18) shows, in order to incentivize positive investment, the ratio between Z^x and Z^y must be less than $-1/\rho$. The two bottom plots in panel (a) show that Z^x jumps to a negative value and Z^y drops significantly at x^*. This ensures $Z^x/Z^y < -1/\rho$, so that the contract incentivizes positive environmental investment. Meanwhile, the gross contract sensitivity to cash flow $Z^e = Z^y + \lambda Z^x$ decreases at x^* and the agent's optimal effort drops as a consequence. When x keeps increasing beyond x^*, the contract financial (environmental) performance sensitivity decreases (increases). This reflects the fact that a significantly greater environmental investment can be motivated even if the magnitude of the environmental sensitivity declines (becomes less negative). In summary, in order to incentivize investment in the environment, the principal penalizes the agent for local pollution and simultaneously reduces the standard financial contract sensitivity, so that the agent becomes less sensitive to cash flow reductions resulting from environmental investment.

Panel (b) of Figure A-1 presents the stationary density of the local pollution intensity x. When x is very large, the pollution reduction stemming from environmental investment exceeds the emissions from production due to greater investment and lower productivity. This generates a negative expected growth in the local pollution intensity and its stationary distribution. Panel (b) presents the 0.1%-quantile to 99.9%-quantile of the stationary distribution, covering a range of [0.28, 0.39] in x. In this range, panel (a) shows that the principal always contracts on pollution and that environmental investment is always positive. The background pollution intensity, the mean of the stationary distribution of x, is 0.323 in this case.

Figure A-2 shows the impact of the pollution intensity λ. Consider first the case where productivity does not depend on pollution, i.e., $p \equiv A$. Endogenous variables are then insensitive to pollution x. The solid black lines in the upper left quadrant in panel (a) show that the agent's optimal effort decreases when λ increases. In this instance, each unit of production generates additional local pollution, which reduces the principal's welfare. She therefore reduces the contract sensitivity to cash flow as λ increases, as shown in the lower left quadrant of panel (a). The upper and lower quadrants in panel (a) show that the contract sensitivity to pollution and the environmental investment are both zero. The behavior of the contract sensitivity to cash flow is consistent with the evidence reported in Yu et al. (2022).

When productivity deteriorates with pollution, panel (a) of Figure A-2 shows that the threshold x^* decreases as λ increases, so that the principal starts to incentivize environmental investment at a lower value of local pollution. For a larger λ, the reduction in the contract sensitivity to cash flow Z^y is larger (lower left quadrant in panel (a)) and the magnitude of the contract sensitivity to local pollution Z^x is lower (lower right quadrant in panel (a)). The resulting ratios of contract sensitivities $Z^i = Z^x/Z^y$ increase in λ, so that (18) implies that the corresponding environmental investments increase as well.

Panel (b) shows the impact of λ on the stationary distribution of x. When λ is larger, production generates more pollution, even after mitigation measures incentivized by the optimal contract, so that the distribution of x shifts to the right, resulting in a larger value of the background pollution intensity $\overline{x}$. Panel (b) shows that most of the mass of the stationary distribution locates in the region where both contracting on pollution and environmental investment are present.

Figure A-3 illustrates the impact of the efficacy ρ of environmental investment. Note first, that in contrast to the previous case, the optimal effort, optimal investment and the contract sensitivity coefficients are not affected if $p \equiv A$. Moreover, the contract pollution sensitivity and optimal investment in mitigation are both null. Pollution, in this case, does not affect productivity, so neither the agent nor the principal alter their behaviors. Second, there are substantial effects if p depends on pollution. In this instance, as ρ increases, environmental investment becomes more efficient in reducing local pollution. Panel (a) shows that the threshold x^* increases with ρ and that the principal starts to reduce the contract sensitivity to cash flow and employs the contract sensitivity to pollution to incentivize environmental investment at higher values of the local pollution intensity. As ρ increases, the reduction in the contract sensitivity to cash flow is lower above x^*, but the magnitude of the contract sensitivity to pollution increases, resulting in roughly the same ratio $Z^i = Z^x/Z^y$ and environmental investment. But even with a similar environmental investment, the more efficient pollution reducing technology has a strong impact on the stationary distribution of x. As panel (b) shows, the stationary distribution of x dramatically shifts to the left as ρ increases, resulting in a lower likelihood of high pollution intensities.

When the principal's concern for the environment increases (i.e., m increases), Figure A-4 panel (a) shows that the threshold x^* decreases, so that the principal reduces the contract sensitivity to cash flow and incentivizes environmental investment at lower values of the local pollution intensity. As a result, the stationary distribution of x shifts to the left and the likelihood of background pollution intensities beyond certain levels decreases, as panel (b) illustrates. As m increases, the lower two quadrants of panel (a) show that the quantitative differences in contract sensitivities are small, when x exceeds the threshold x^*. Nevertheless, the ratio Z^x/Z^y decreases in m, resulting in an increasing environmental investment, as shown in the top right quadrant of panel (a). Although not displayed, similar effects arise when the agent becomes more sensitive to pollution.

When the principal puts more weight on the background pollution intensity $\overline{x}$ and less weight on the local pollution intensity x, i.e., when ζ decreases, the impact on the optimal contract is equivalent to a reduction in m. The resulting environmental investment decreases and the distribution of pollution shifts to the right. A policy implication of this result is that firms need to be held more accountable for their own emissions in order to reduce aggregate pollution.

4 Environmental contracting, welfare and equilibrium

In the previous section, we examined the optimal contract when the principal fully recognizes the damage of pollution on production. The optimal contract in this case depends on both firm cash flow and pollution, balancing the incentives pertaining to the firm's financial performance and its environmental impact. In this section, we examine the welfare and equilibrium impact of contracting on firm pollution. To this end, we assume that both the agent and the principal still have environmental concerns with utility specifications as described in (9) and (10). The principal observes both the local and the background pollution, but *fails* to recognize the damaging impact of pollution on production, i.e., $p \equiv p_0$ for a constant p_0. Such a principal, with incorrect beliefs about the effects of pollution on productivity, is called a *misperceiving principal*. In contrast, the agent, who manages the production process, fully recognizes that p deteriorates with the level of pollution and makes his effort and environmental investment decisions accordingly.

Throughout the section, we assume that

$$\frac{\lambda}{r}\left(\frac{\gamma_x}{\gamma_c}\frac{\ell}{K} + m\zeta\right) < 1 \quad \text{and} \quad \frac{\rho}{r}\left(\frac{\gamma_x}{\gamma_c}\frac{\ell}{K} + m\zeta\right) < 1 \tag{36}$$

so that the marginal benefit of production is larger than its pollution impact, but the environmental concern is not sufficiently strong for the misperceiving principal to contract on pollution in order to incentivize environmental investment. With $p \equiv p_0$, the optimal contract sensitivities, obtained at the end of Section 2.2, are

$$Z^{S,y*} = \frac{1 - \frac{\lambda}{r}\left(\frac{\gamma_x}{\gamma_c}\frac{\ell}{K} + m\zeta\right)}{1 + \frac{\gamma_c r \theta^e K (\sigma^y)^2}{p_0^2}} \quad \text{and} \quad Z^{S,x*} = 0. \tag{37}$$

From the misperceiving principal's point of view, the *recommended* effort and investment policies are

$$e^{S,*} = \frac{p_0}{\theta^e} Z^{S,y*} \qquad \text{and} \qquad i^{S,*} = 0. \tag{38}$$

The optimal compensation stream for the misperceiving principal is presented in the following Lemma.

Lemma 5 *If the misperceiving principal believes that $p \equiv p_0$, a constant, the optimal compensation is*

$$\begin{aligned} dI^S = \Big[&- \frac{1}{\gamma_c} \log r + r\mathcal{U}_0 + r\Big(\frac{\gamma_c r}{2}(Z^{S,y*} K \sigma^y)^2 - Z^{S,y*} p_0 (\mu + e^{S,*}) K\Big) t \\ &+ g(e^{S,*}, x_t, \overline{x}_t, K) + \frac{\gamma_x}{\gamma_c}\big(\ell x_t + (1-\ell)\overline{x}_t\big) + r Z^{S,y*} Y_t \Big] dt, \end{aligned} \tag{39}$$

where $\mathcal{U}_0$ is the agent's reservation certainty equivalent. If p were truly p_0, the agent would not save and would consume all his compensation net his effort cost.

Comparing to the optimal compensation stream dI^* in (25), the optimal contract dI^S for the misperceiving principal does not contract on firm's pollution X, except compensating for the environmental impact $\frac{\gamma_x}{\gamma_c}\big(\ell x + (1-\ell)\overline{x}\big)$ on the agent. The contract sensitivity to firm cash flow $Z^{S,y*}$ is a constant. Hence the compensation stream dI^S comprises a deterministic component which is linear in the agent's tenure in the firm, and a variable component which is linear in the cumulative cash flow Y.

Recognizing that the firm's productivity actually deteriorates with pollution, the agent's optimal policies are different from the principal's recommendations when receiving the compensation stream in Lemma 5. Given the consumption stream dI^S in (39), the agent determines his optimal policy by solving the following optimization problem

$$\sup_{e,i,c} \mathbb{E}\Big[\int_0^\infty e^{-rt} u(c_t, x_t, \overline{x}_t) dt\Big], \tag{40}$$

subject to (4), (5), and (8), with dI therein replaced by dI^S. Introduce the agent's certainty equivalent $\mathcal{U}$ via (12). The following result summarizes the agent's optimal strategies for the given compensation stream dI^S.

Lemma 6 *For the given compensation stream dI^S in* (39)*, the agent's continuation certainty equivalent is given by*

$$\mathcal{U}_t = \mathcal{U}(X_t) + \Big(\frac{\gamma_c r}{2}\big(K\sigma^y Z^{S,y*}\big)^2 - p_0(\mu + e^{S,*})KZ^{S,y*}\Big)t + Z^{S,y*}Y_t,$$

where the function $\mathcal{U}$ satisfies

$$\begin{aligned} r\mathcal{U} =& \frac{1}{2}(\sigma^x K)^2 \partial^2_{XX}\mathcal{U} + \big(\lambda p(\mu + e^*)K - \rho i^* K\big)\partial_X \mathcal{U} \\ &+ \big(p(\mu + e^*)K - i^*K - k(i^*, K) - p_0(\mu + e^{S,*})K\big)Z^{S,y*} \\ &+ g(e^{S,*}, K) - g(e^*, K) - \frac{\gamma_c r}{2}\big(K\sigma^x \partial_X \mathcal{U}\big)^2. \end{aligned} \tag{41}$$

and $Z^{S,y*}$ and $e^{S,*}$ are constants given in (37) *and* (38)*. The agent's optimal effort e^* and environmental investment i^* are*

$$e^* = Z^e \frac{p(x, \overline{x})}{\theta^e} \quad \text{and} \quad i^* = \begin{cases} \frac{-\rho Z^i - 1}{\theta^i}, & -\rho Z^i_t > 1 \\ 0, & \text{otherwise} \end{cases}, \tag{42}$$

with $Z^e = Z^{S,y*} + \lambda \partial_X \mathcal{U}$ and $Z^i = \frac{\partial_X \mathcal{U}}{Z^{S,y*}}$.

For the given compensation stream dI^S, the agent's continuation certainty equivalent depends linearly on the firm's cash flow Y with sensitivity $Z^{S,y*}$, the contract's sensitivity coefficient specified by the misperceiving principal. Even though the compensation stream dI^S does not depend explicitly on the firm's pollution X, except for a pollution pay $\frac{\gamma_x}{\gamma_c}\big(\ell x_t + (1-\ell)\overline{x}_t\big)$ to rebate the direct impact of pollution on the agent's utility, the agent may still invest in pollution abatement because he fully recognizes the adverse impact of pollution on productivity, which reduces the firm's cash flow hence his compensation. When the marginal (certainty-equivalent) cost of pollution, i.e., $-\partial_X \mathcal{U}$, is higher than $Z^{S,y*}/\rho$, the agent invests in the environment to manage the adverse impact of pollution on productivity. The marginal cost of pollution also reduces the gross contract sensitivity to cash flow Z^e, so that it is lower than the contract sensitivity $Z^{S,y*}$ specified by the principal. Moreover, the productivity p

decreases with pollution, hence it is smaller than the value p_0 perceived by the principal. These two effects combined reduce the agent's effort so that his optimal effort e^* is less than the principal's recommendation $e^{S,*}$.

Having obtained the agent's optimal response to the compensation stream dI^S, we next examine the impact on the principal's welfare and the background environmental pollution, if the principal offers the contract dI^S instead of the optimal contract in Section 2. To this end, we fix the agent's reservation certainty equivalent $\mathcal{U}_0 = \frac{1}{r\gamma_c}\log r - \frac{\gamma_x}{\gamma_c r}(1-\ell)\overline{x}$, corresponding to an outside option without consumption and local pollution, but still with background pollution.

Given the compensation stream dI^S, the misperceiving principal's value is

$$\mathbb{E}\Big[\int_0^\infty e^{-rt}\big(dY_t - dI_t^S - mK(\zeta x_t + (1-\zeta)\overline{x}_t)dt\big)\Big],$$

where Y and X follow (4) and (5), respectively, evaluated at the agent's optimal policies e^* and i^* given in (42). The principal's value is characterized next.

Proposition 7 *Given the compensation stream dI^S and a constant background pollution intensity $\overline{x}$, the misperceiving principal's value function*

$$V_t = \mathbb{E}_t\Big[\int_t^\infty e^{-r(s-t)}\big(dY_s - dI_s^S - mK(\zeta x_s + (1-\zeta)\overline{x})ds\big)\Big]$$

is characterized by

$$V_t = \mathcal{V}(X_t) - \Big(\frac{\gamma_c r}{2}\big(K\sigma^y Z^{S,y*}\big)^2 - p_0(\mu + e^{S,*})KZ^{S,y*}\Big)t - Z^{S,y*}Y_t,$$

where the function $\mathcal{V}$ satisfies

$$
\begin{aligned}
r\mathcal{V} =& \frac{1}{2}(\sigma^x K)^2 \partial^2_{XX}\mathcal{V} + \big(\lambda p(\mu + e^*)K - \rho i^* K\big)\partial_X \mathcal{V} \\
& \underbrace{-\frac{\gamma_c r}{2}\big(K\sigma^y Z^{S,y*}\big)^2 - g(e^{S,*}, x, \overline{x}, K) + p_0(\mu + e^{S,*})KZ^{S,y*} - \frac{\gamma_x}{\gamma_c}\ell x}_{\textit{Cost of contract perceived by principal}} \\
& \underbrace{+(1 - Z^{S,y*})\big(p(\mu + e^*)K - i^*K - k(i^*, K)\big)}_{\textit{Share of cash flow to principal}} \\
& \underbrace{-mK\big(\zeta x + (1-\zeta)\overline{x}\big)}_{\textit{Environmental cost to principal}} .
\end{aligned} \tag{43}
$$

In the previous equation, $p = p(x, \overline{x})$, e^*, i^* *are given in* (42), *and* $e^{S,*}$ *and* $Z^{S,y*}$ *are from* (37) *and* (38).

On the right-hand side of (43), the group of terms in the second line represents a net cost of the contract as perceived by the principal. This perceived net cost includes a risk-bearing compensation to the agent, a rebate for the agent's effort cost based on the recommended effort, a compensation for the environmental cost borne by the agent, and is net of the expected cash flow collected by the agent in the principal's perception. The group of terms in the third line represents the share of cash flow collected by the principal. The last term on the right-hand side of (43) is the environmental cost to the principal.

To examine the impact of pollution contacting on the principal's welfare, we consider a firm whose principal erroneously believes that $p \equiv p_0$ and utilizes the compensation stream dI^S in (39), meanwhile all other firms in the continuum recognize the adverse impact of pollution on production and employ the optimal contract in Section 2. The equilibrium background pollution intensity $\overline{x}$ is the same as the one in Section 2, because one firm in a continuum of firms with homogeneous sizes does not impact the aggregate background pollution intensity.

Using the calibration in Section 3, Figure A-5 illustrates the impact when one firm, with a misperceiving principal, deviates from the optimal contract in Section 2 to employ the contract dI^S, which does not contract on the firm's pollution directly. We call this contract the *standard contract*. The upper left quadrant in panel (a) shows that the agent's optimal effort decreases with the local pollution intensity. However, the agent's effort under the optimal contract jumps down when the local pollution intensity exceeds a threshold. Under the standard contract, the agent's effort also decreases with the local pollution intensity x, because the marginal cost of pollution $-\partial_X \mathcal{U}$ increases with x (see lower right quadrant in panel (a)), reducing the gross certainty-equivalent sensitivity to cash flow, but it does not jump down. The upper right quadrant of panel (a) shows that environmental investment is more aggressive under the optimal contract, since investment becomes positive when x exceeds the threshold x^*. Meanwhile, the environmental investment under the standard contract can still be positive, but only kicks in when the local pollution is sufficiently severe and the marginal cost of pollution exceeds a higher threshold. Panel (b) in Figure A-5 compares the principals' welfare under their respective optimal contracts. Pollution contracting in the optimal contract improves the principal's welfare, for all pollution levels, compared to the standard contract. Recognition of the adverse impact of pollution on the firm's production motivates the principal to use the optimal contract, which incentivizes a more aggressive abatement strategy and mitigates the impact of pollution on production.

If *all* firms in the economy erroneously believes that $p \equiv p_0$ and employ the standard contract dI^S, the aggregated background pollution deteriorates. Panel (c) of Figure A-6 shows that the background pollution intensity increases by 62.2% compared to the situation where all firms employ the optimal contract. Equivalently, if all firms in the economy correctly recognize the adverse impact of pollution on productivity and all migrate from the standard contract to the optimal contract with pollution contracting, then background pollution intensity will decrease by 38.4%. Comparison between the red dotted line in the upper right quadrant of panel (a) in Figure A-6 and its counterpart in Figure A-5 shows that more severe background pollution motivates a more aggressive abatement strategy. However, comparison between the red dotted line in panel (b) of Figure A-6 and its counterpart in Figure A-5 reveals that each misperceiving principal's value decreases in the former case, due to the deterioration of the background pollution in the environment. Moreover, the agent's reservation utility also decreases with the background pollution when all principals misperceive and employ the standard contract.

In summary, comparisons in this section show that contracting on pollution not only improves the welfare of the agent and the principal, but also benefits the environment when it is adopted by all firms.

5 Conclusion

In this paper, we examined optimal dynamic contracts in a principal-agent relationship when pollution affects the productivity of the firm and the agent can take mitigating actions by investing in pollution-abating activities. We show the optimal contract rewards for environmental performance as well as financial performance. Numerical results, in a calibrated version of the model, show that the optimal effort level decreases with the level of pollution and the optimal environmental investment increases with it once a critical pollution threshold is reached. The agent's (positive) certainty-equivalent sensitivity to financial performance decreases past that level and its (negative) sensitivity to environmental performance increases, except at the threshold level where it jumps down. We also derive the stationary distribution of aggregate pollution in an economy with a continuum of identical firms and examine its properties. The emission rate, the environmental investment efficiency, and the principal's environmental concerns are shown to have significant impacts on the contractual terms, the optimal policies and the stationary distribution. Finally, we show that rewarding for environmental performance, in addition to financial performance, improves the welfare of the principal and the agent in the stationary equilibrium.

The results in the paper provide guidelines about how to structure contracts when pollution and emissions are a concern. They show that a simple amendment of the typical performance-based contract can be an effective tool to mitigate the impact of emissions and improve the welfare of economic agents. A critical aspect of the theory developed is the internalization of the costs of pollution by the agent or the principal. The first, who is working on company grounds hence exposed to the firm's emissions, directly bears health consequences and therefore ought to care about his immediate environment. As for the second, who might be living in distant suburbs and be less directly exposed, it takes a realization of the effects of pollution on her employees and a concern for their well-being, as well as a recognition of the damaging effects of pollution on productivity. The public dissemination of information pertaining to the firm's emissions, the ambient pollution in her geographical area and the impact of pollution on productivity, may be conducive to raise such awareness. Each of the first two aspects, in combination with the third one, will be enough to justify contracts rewarding on environmental performance.

While contracts may help to address some of the issues regarding pollution, the more important question is whether they will have enough of an effect to solve the bigger challenge pertaining to climate change. Scientific evidence shows that global warming has accelerated over the past few decades and is nearing thresholds that are thought to be tipping points for the climate. Calls for action recommending a $+1.5^{o}$C limit to further temperature increases, as per the Paris Agreement at COP21, are likely to require drastic reductions in anthropomorphic emissions. Achieving that goal through contracts seems unlikely since existing contracts would have to be rewritten on a global scale and the relevant climate concerns and effects internalized by all parties involved. Regulations imposing higher carbon taxes, placing outright limits on emissions or mandating contracts rewarding for environmental performance may all be needed to address the global challenge.

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A Appendix

Proof of Lemma 2

Consider the process

$$\widetilde{U}_t = e^{-rt}U_t^a + \int_0^t e^{-rs}u(c_s, x_s, \overline{x}_s)ds.$$

It follows from the dynamic programming principle that $\widetilde{U}$ is a supermartingale for an agent's arbitrary strategy (e, i, c) and is a martingale for the optimal strategy (e^*, i^*, c^*). Combining (12), (13), and Itô's formula, the drift of $\widetilde{U}$, divided throughout by $-r\gamma_c e^{-rt}U_t^a$, is

$$\Big[\frac{1}{\gamma_c} + rW_t - c_t - g(e_t, x_t, \overline{x}_t, K) + Z^y\big(e_t p(x_t, \overline{x}_t)K - i_t K - k(i_t, K)\big) + Z^x\big(\lambda e_t p(x_t, \overline{x}_t)K - \rho i_t K\big)$$
$$- \frac{r\gamma_c}{2}\big[(Z^y K\sigma^y)^2 + (Z^x K\sigma^x)^2\big] - \frac{1}{r\gamma_c}e^{-\gamma_c c_t + \gamma_x(x_t + \ell\overline{x}_t + \gamma_c rW_t + \gamma_c r\mathcal{U}_t}\Big]dt + dH_t + dI_t.$$

This drift is non-positive for all strategies and zero for the optimal one. Therefore

$$\begin{aligned} dH_t = \min_{e,i,c}\Big\{ & -\frac{1}{\gamma_c} - rW_t + c_t + g(e_t, x_t, \overline{x}_t, K) - \\ & Z^y K\big(e_t p(x_t, \overline{x}_t) - i_t - k(i_t)\big) - Z^x K\big(\lambda e_t p(x_t, \overline{x}_t) - \rho i_t\big) \\ & + \frac{r\gamma_c}{2}\big[(Z^y\sigma^y)^2 + (Z^x\sigma^x)^2\big] + \frac{1}{r\gamma_c}K^2 e^{-\gamma_c c_t + \gamma_x(x_t + \ell\overline{x}_t) + \gamma_c rW_t + \gamma_c r\mathcal{U}_t}\Big\}dt - dI_t. \end{aligned}$$

The optimal (e^*, i^*, c^*) in (16), (17), and (18) are obtained from the first order conditions to this problem. They are also the minimizer for the minimization problem above due to the convexity of g, k, and the exponential function. Plugging the expressions of (e^*, i^*, c^*) into the previous expression of dH_t and (13), we obtain the dynamics of $\mathcal{U}$ in (14).

Using the standard verification argument, we can show that (e^*, i^*, c^*) is the optimal policy for the agent.

Proof of Proposition 3

For a given constant background pollution intensity $\overline{x}$, the local pollution intensity x follows

$$dx_t = \big(\lambda p(x_t, \overline{x}(\mu + e_t^*) - \rho i_t^*\big)dt + \sigma^x dB_t^x.$$

Consider x as the state variable for the problem (22), the HJB equation (23) follows from the dynamic programming principle.

To derive the optimal compensation stream in (25), consider the compensation stream

$$dI_t^* = \Big[-\frac{1}{\gamma_c} \log r + g(e_t^*, x_t, \overline{x}, K) + \frac{\gamma_x}{\gamma_c} \big(\ell x_t + (1-\ell)\overline{x}\big) + r\mathcal{U}_t \Big] dt, \tag{A.1}$$

where $\mathcal{U}$ is the agent's certainty equivalent introduced in (12). Applying Lemma 2, we obtain the dynamics of $\mathcal{U}$ in (14) and agent's optimal strategies in (16), (17), and (18). In particular, plugging (16) and (A.1) into (8), we obtain that $dW \equiv 0$, i.e., the agent optimally does not save. Moreover, plugging (A.1) into (14), we obtain

$$d\mathcal{U}_t = \frac{\gamma_c r}{2} K^2 \big((Z^{y*}\sigma^y)^2 + (Z^{x*}\sigma^x)^2 \big) dt + Z^{y*} K \sigma^y dB^y + Z^{x*} K \sigma^x dB^x.$$

Integrating the previous dynamics in time, combining with (4), (5), and plugging the resulting expression into (A.1), we obtain the optimal compensation stream in (25).

Proof of Corollary 4

When $p \equiv p_0$ and $\theta^e \equiv \theta_0^e$, the HJB equation (23) admits an explicit solution

$$V(x) = -\frac{1}{r} \big(\frac{\gamma_x}{\gamma_c} \ell + mK\zeta \big) x + \text{constant}.$$

Plugging the previous expression into (23), we obtain (Z^e, Z^i) as the maximizer of (26) and the constant term is identified by matching constant terms on both sides of (23).

Proof of Lemma 5

The proof is similar to that for Proposition 3. We present it here for completeness. Given the contract sensitivities $Z^{S,y*}$ and $Z^{S,x*}$ given in (37), consider a compensation stream given by

$$dI_t^S = \left(-\frac{1}{\gamma_c} \log r + g(e^{S,*}, K) + \frac{\gamma_x}{\gamma_c} \big(\ell x_t + (1-\ell)\overline{x}\big) + r\mathcal{U}_t \right) dt, \tag{A.2}$$

where $e^{S,*}$ is given in (38) and $\mathcal{U}$ is the agent's continuation certainty equivalent. Plug the previous equation into (14) with $Z^x = 0$ and $Z^y = Z^{S,y*}$. Then $\mathcal{U}$ follows

$$d\mathcal{U}_t = \frac{\gamma_c r}{2}(Z^{S,y*}K\sigma^y)^2 dt + Z^{S,y*}K\sigma^y dB^y. \tag{A.3}$$

If p were truly p_0, Lemma 2 implies that the agent's optimal consumption rate would be given by (16) and the optimal effort and investment would be $e^{S,*}$ and $i^{S,*}$. Plugging (16) and (A.2) into the agent's wealth dynamics (8), we obtain $dW_t \equiv 0$, therefore, the agent does not save.

The principal assumes that the agent follows the recommended policies $e^{S,*}$ and $i^{S,*}$. Therefore, in the principal's view, the firm's cash flow follows

$$dY_t = p_0(\mu + e^{S,*})K dt + \sigma^y K dB_t^y. \tag{A.4}$$

Combining (A.3) and (A.4), integrating in time, and plugging into (A.2), we obtain (39).

Proof of Lemma 6

Define the agent's continuation certainty equivalent $\mathcal{U}$ by

$$U_t^a = \sup_{e,i,c} \mathbb{E}_t \Big[\int_t^{\infty} e^{-r(s-t)} u(c_s, x_s, \overline{x}) ds \Big] = -e^{-\gamma_c r(W_t + \mathcal{U}_t)}.$$

The process $\mathcal{U}$ admits the decomposition

$$d\mathcal{U}_t = dH_t + Z_t^y dY_t + Z_t^x dX_t$$

with H, Z^x, and Z^y to be determined. Following the proof of Lemma 2 and using the form of dI^S in (39), we obtain

$$
\begin{aligned}
d\mathcal{U}_t =& \{r\mathcal{U}_t + \Phi_t\}\, dt \\
&+ Z_t^y \left[dY_t - (p_t(\mu + e_t^*)K - i_t^* K - k(i_t^*, K))\, dt\right] + Z_t^x \left[dX_t - (\lambda p_t(\mu + e_t^*)K - \rho i_t^* K)\, dt\right]
\end{aligned} \tag{A.5}
$$

$$
\begin{aligned}
\Phi_t =& g(e_t^*, K) - g(e_t^{S,*}, K) - r\Big(\frac{\gamma_c r}{2}(Z^{S,y*} K\sigma^y)^2 - Z^{S,y*} p_0(\mu + e^{S,*})K\Big)t - rZ^{S,y*} Y_t \\
&+ \frac{\gamma_c r}{2} K^2 \left[(Z_t^y \sigma^y)^2 + (Z_t^x \sigma^x)^2\right]
\end{aligned} \tag{A.6}
$$

where the agent's optimal consumption c^*, effort e^*, and environmental investment i^* are given by (16), (17), (18), the recommended effort $e^{S,*}$ and the contract sensitivity $Z^{S,y*}$ are given in (38) and (37). In particular, e^* and i^* depend on $Z^e = Z^y + \lambda Z^x$ and $Z^i = \frac{Z^x}{Z^y}$.

To remove the time dependency in (A.6), we introduce the new variable

$$
\mathcal{Y}_t = Y_t + \Big(\frac{\gamma_c r}{2}(K\sigma^y)^2 Z^{S,y*} - p_0(\mu + e^{S,*})K\Big)t. \tag{A.7}
$$

Consider $\mathcal{Y}$ and X as two state variables for the agent's problem, $\mathcal{U}_t = \widetilde{\mathcal{U}}(\mathcal{Y}_t, X_t)$ for a function $\widetilde{\mathcal{U}}$. Next, apply Itô's formula to $\widetilde{\mathcal{U}}(\mathcal{Y}_t, X_t)$ and compare with the right-hand side of (A.5). We obtain

$$
Z^y = \partial_{\mathcal{Y}} \widetilde{\mathcal{U}}, \quad Z^x = \partial_X \widetilde{\mathcal{U}}, \quad \text{and}
$$

$$
\begin{aligned}
&\frac{1}{2}(\sigma^y K)^2 \partial^2_{\mathcal{Y}\mathcal{Y}} \widetilde{\mathcal{U}} + \frac{1}{2}(\sigma^x K)^2 \partial^2_{XX} \widetilde{\mathcal{U}} \\
&+ \Big(\frac{\gamma_c r}{2}(K\sigma^y)^2 Z^{S,y*} - p_0(\mu + e^{S,*})K + p(\mu + e^*)K - i^* K - k(i^*, K)\Big)\, \partial_{\mathcal{Y}} \widetilde{\mathcal{U}} \\
&+ (\lambda p(\mu + e^*)K - \rho i^* K)\, \partial_X \widetilde{\mathcal{U}} \\
=& r\widetilde{\mathcal{U}} + g(e^*, K) - g(e^{S,*}, K) - rZ^{S,y*}\mathcal{Y} + \frac{\gamma_c r}{2} K^2 \left[(\sigma^y \partial_{\mathcal{Y}} \widetilde{\mathcal{U}})^2 + (\sigma^x \partial_X \widetilde{\mathcal{U}})^2\right].
\end{aligned} \tag{A.8}
$$

The previous equation admits a solution

$$
\widetilde{\mathcal{U}}(\mathcal{Y}, X) = \mathcal{U}(X) + Z^{S,y*}\mathcal{Y},
$$

for some function $\mathcal{U}$. Hence $\partial_{\mathcal{Y}} \widetilde{\mathcal{U}} = Z^{S,y*}$, $\partial_X \widetilde{U} = \partial_X \mathcal{U}$, and the previous equation is reduced to the equation (41). Using $Z^y = Z^{S,y*}$ and $Z^x = \partial_x \mathcal{U}$, we also obtain e^* and i^* from (17) and (18).

Proof of Proposition 7

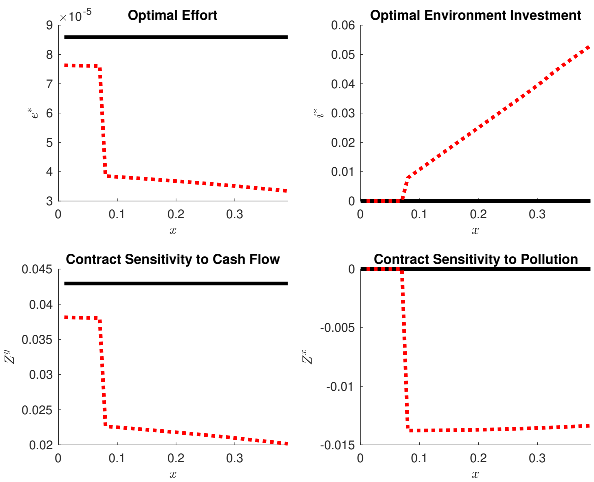
Consider X and $\mathcal{Y}$ in (A.7) as two state variables for the principal's value function V. Then V satisfies the following differential equation

$$
\begin{aligned}
rV =& \frac{1}{2}(\sigma^x K)^2 \partial^2_{XX} V + \frac{1}{2}(\sigma^y K)^2 \partial^2_{\mathcal{Y}\mathcal{Y}} V + \big(\lambda p(\mu + e^*)K - \rho i^* K\big)\partial_X V \\
&+ \Big(\frac{\gamma_c r}{2}(\sigma^y K)^2 Z^{S,y*} - p_0(\mu + e^{S,*})K + p(\mu + e^*)K - i^* K - k(i^*, K)\Big)\partial_{\mathcal{Y}} V \\
&+ p(\mu + e^*)K - i^* K - k(i^*, K) - g(e^{S,*}, K) - rZ^{S,y*}\mathcal{Y} - \frac{\gamma_x}{\gamma_c}\ell x - mK(\zeta x + (1-\zeta)\overline{x}).
\end{aligned}
$$

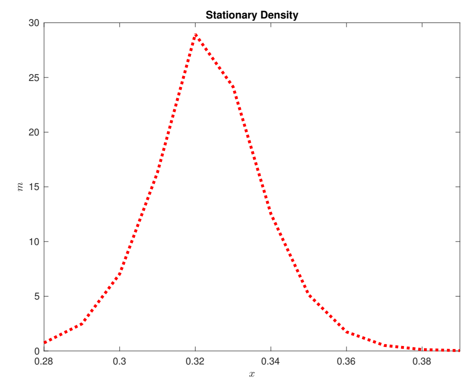
This equation admits a solution

$$
V(X, \mathcal{Y}) = \mathcal{V}(X) - Z^{S,y*}\mathcal{Y}.
$$

Plugging this decomposition into the previous equation for V, we obtain the equation satisfied by $\mathcal{V}$ in (43).



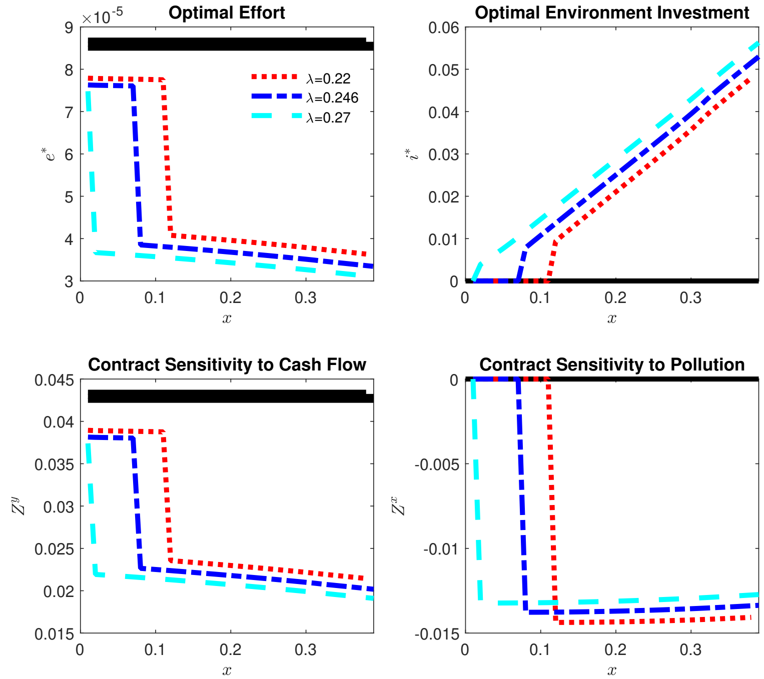
(a) Agent's optimal effort, environmental investment, and optimal contract sensitivities to cash flow and pollution



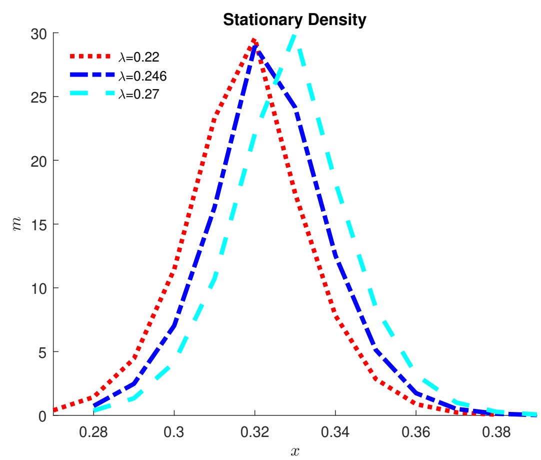
(b) Stationary distributions

Figure A-1: **Optimal policies, contract, and stationary density of local pollution intensity in the calibrated**

Notes: Panel (a) reports the agent's optimal effort e^*, the optimal environmental investment i^*, and the optimal contract sensitivities to cash flow Z^y and to local pollution Z^x. The black solid lines represent the corresponding quantities when the productivity does not depend on the local nor background pollution intensities, i.e., $p \equiv A$. Panel (b) shows the stationary distribution of the local pollution intensity between the 0.1%-quantile and the 99.9%-quantile of the stationary distribution. The background pollution intensity in equilibrium is 0.323. Parameters are listed in Table 1.



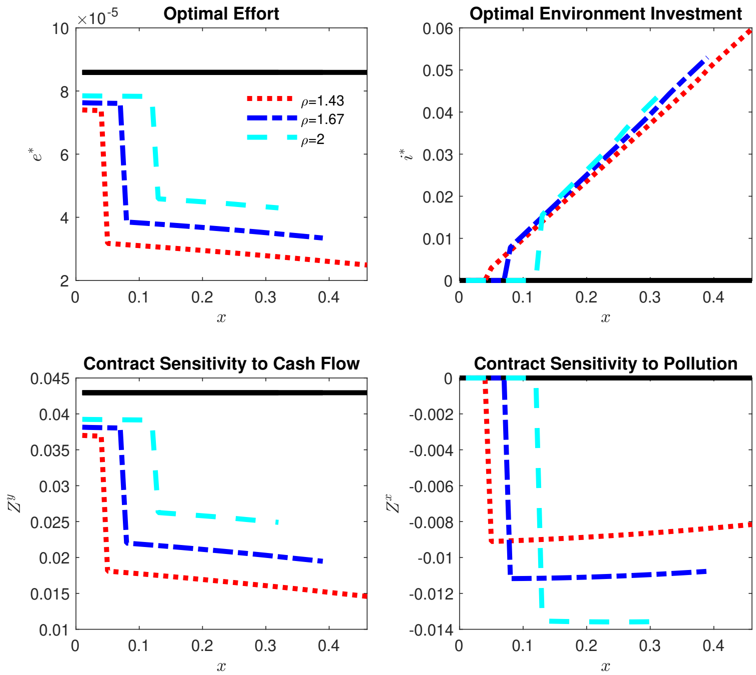
(a) Agent's optimal effort, environmental investment, and optimal contract sensitivities to cash flow and pollution



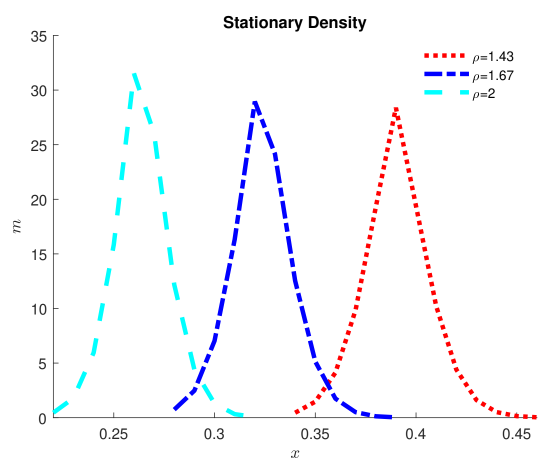
(b) Stationary distributions

Figure A-2: **Impact of production pollution intensity**

Notes: Panel (a) reports the agent's optimal effort e^*, the optimal environmental investment i^*, and the optimal contract sensitivities to cash flow Z^y and to local pollution Z^x for different production pollution coefficients $\lambda = 0.22, 0.246$, and 0.27. The black solid lines represent the corresponding quantities when the productivity does not depend on the local nor background pollution intensities, i.e., $p \equiv A$. Panel (b) shows the stationary distribution of the local pollution intensity between the 0.1%-quantile and the 99.9%-quantile of the stationary distribution. Parameters are listed in Table 1.



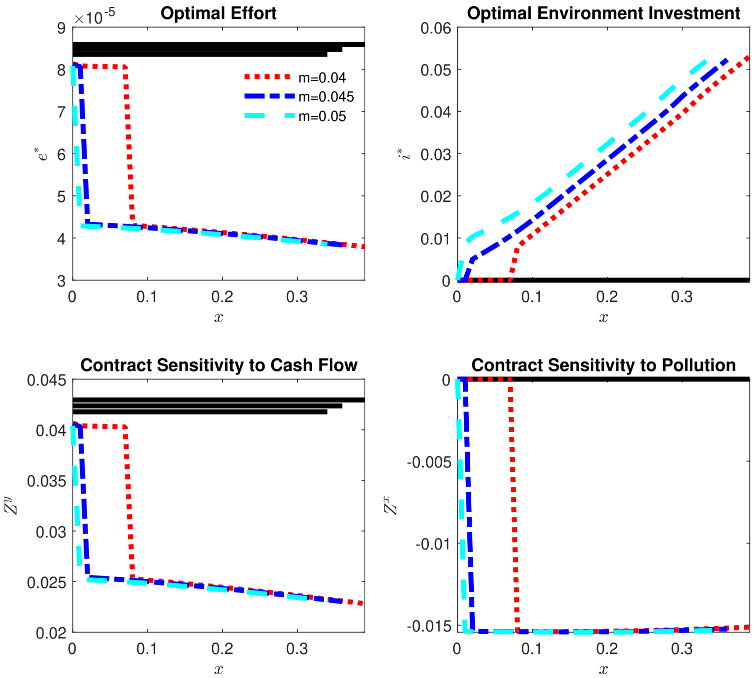
(a) Agent's optimal effort, environmental investment, and optimal contract sensitivities to cash flow and pollution



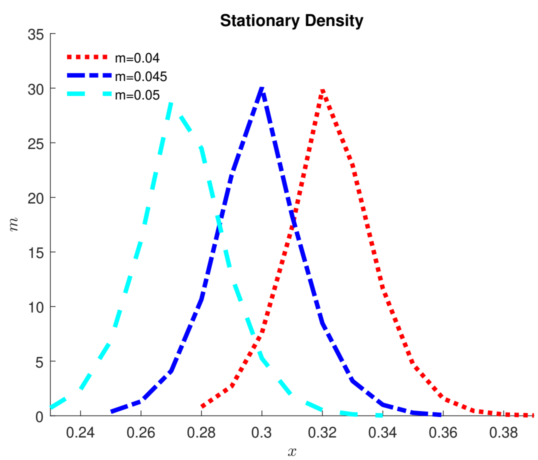
(b) Stationary distributions

Figure A-3: **Impact of environmental investment efficacy**

Notes: Panel (a) reports the agent's optimal effort e^*, the optimal environmental investment i^*, and the optimal contract sensitivities to cash flow Z^y and to local pollution Z^x for different production pollution coefficients $\rho = 1.43, 1.67$, and 2, corresponding to CO2 absorbing technologies with respective costs $700, 600$, and 500 dollars/tons. The black solid lines represent the corresponding quantities when the productivity does not depend on the local nor background pollution intensities, i.e., $p \equiv A$. Panel (b) shows the stationary distribution of the local pollution intensity between the 0.1%-quantile and the 99.9%-quantile of the stationary distribution. Parameters are listed in Table 1.



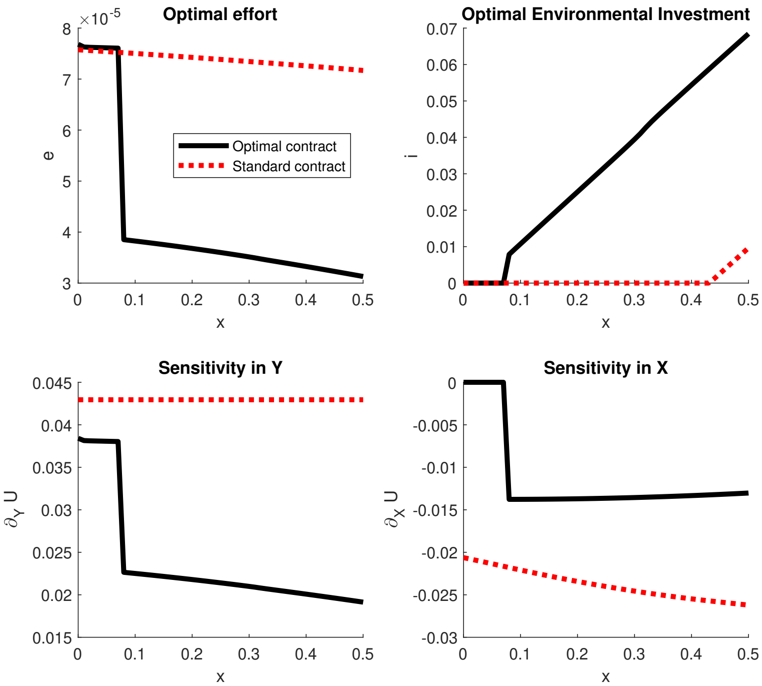
(a) Agent's optimal effort, environmental investment, and optimal contract sensitivities to cash flow and pollution



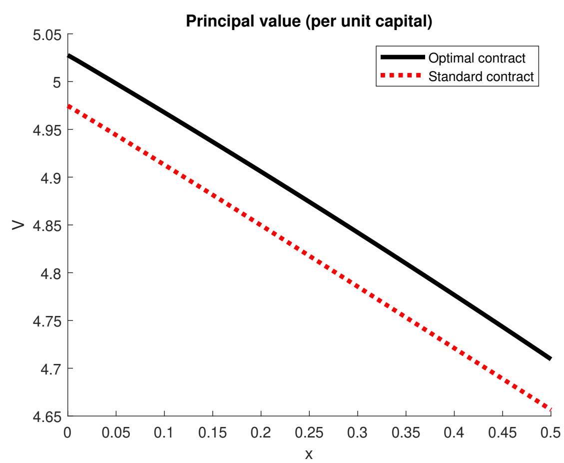
(b) Stationary distributions

Figure A-4: **Impact of principal's environmental concern**

Notes: Panel (a) reports the agent's optimal effort e^*, the optimal environmental investment i^*, and the optimal contract sensitivities to cash flow Z^y and to local pollution Z^x for different production pollution coefficients $m = 0.04, 0.045$, and 0.05. The black solid lines represent the corresponding quantities when the productivity does not depend on the local nor background pollution intensities, i.e., $p \equiv A$. Panel (b) shows the stationary distribution of the local pollution intensity between the 0.1%-quantile and the 99.9%-quantile of the stationary distribution. Parameters are listed in Table 1.



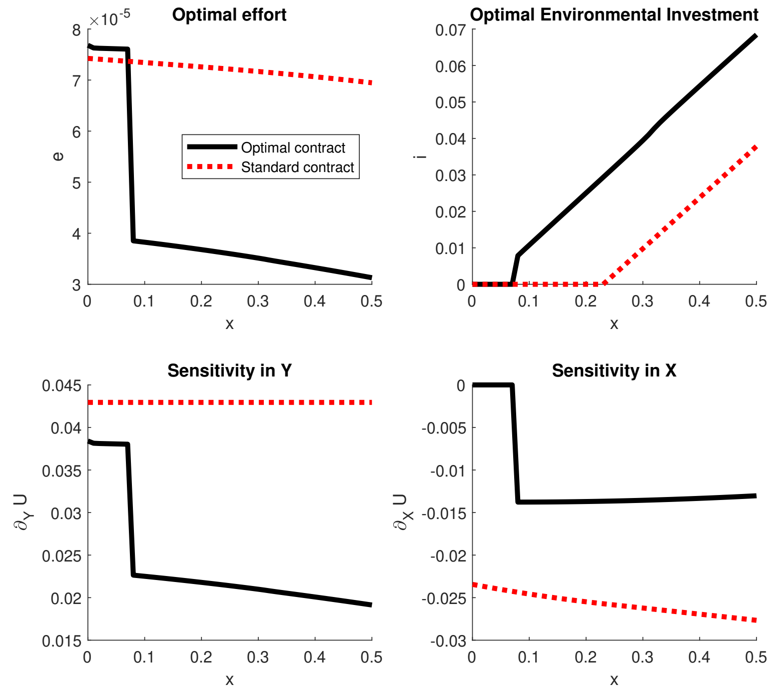
(a) Agent's optimal effort, environmental investment, and sensitivities of agent's certainty equivalent to cash flow and pollution



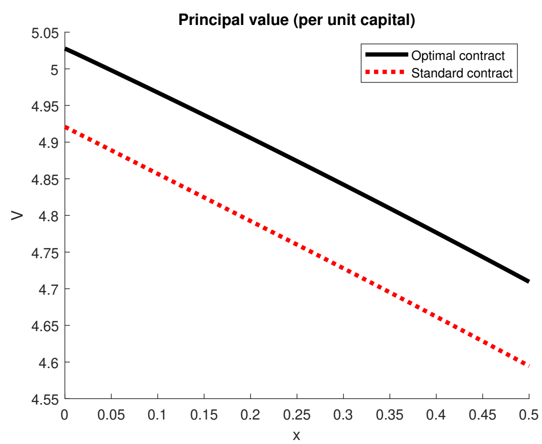
(b) Principal's values in different contracts

Figure A-5: **Impact of different contracts for given background pollution**

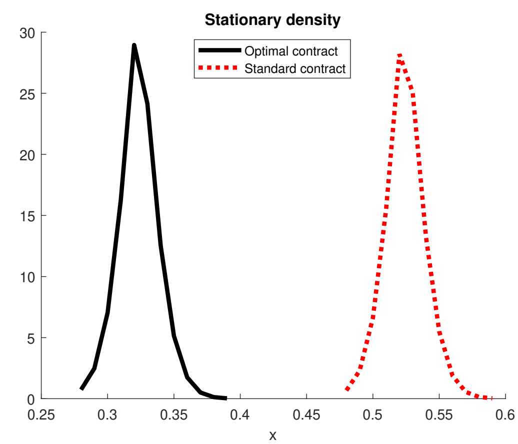
Notes: All black solid lines represent the situation where all firms employ the optimal contract in Section 2. All red dotted lines represent the situation where *one* firm employs the compensation stream dI^S in (39), which we call the standard contract, but all other firms employ the optimal contract in Section 2. In both cases, the background pollution intensity is $\overline{x} = 0.323$, which is the equilibrium background pollution intensity when all firms employ the optimal contract. Panel (a) reports the agent's optimal effort e^*, the optimal environmental investment i^*, and the sensitivities of agent's certainty equivalent under the optimal contract and the standard contract. Panel (b) shows the principal's value in different contracts. Parameters are listed in Table 1.



(a) Agent's optimal effort, environmental investment, and sensitivities of agent's certainty equivalent to cash flow and pollution



(b) Principal's values in different contracts



(c) Stationary distribution

Figure A-6: **Impact of different contracts in equilibrium**

Notes: All black solid lines represent the situation where all firms employ the optimal contract in Section 2. All red dotted lines represent the situation where all firms employ the compensation stream dI^S in (39), which we call the standard contract. Panel (a) reports the agent's optimal effort e^*, the optimal environmental investment i^*, and the sensitivities of agent's certainty equivalent under the optimal contract and the standard contract. Panel (b) shows the principal's value in different contracts. Panel (c) presents the stationary distributions of local pollution intensity: $\overline{x} = 0.323$ for the optimal contract, $\overline{x} = 0.524$ for the standard contract. Parameters are listed in Table 1.